PANEL SELECTION UNDER VETO: A GAME THEORETIC ANALYSIS

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Abstract

We consider a class of selection games where members of a panel, such as a jury, tribunal, or committee, are selected from among a larger pool of individuals by two or more players with potentially non-aligned interests. Each player may exercise a limited number of vetoes against unfavorable panelists, however, at the time a veto is exercised, the players may have incomplete information regarding the favorability of potential replacement panelists. We derive game theory based veto strategies for single-seat and multiple-seat panels where utilities are separable functions of individual panelist utility. We provide numerically derived optimal 'veto thresholds' and we show how these thresholds vary with panel size and with the number of vetoes available to each party. Finally, using computer-simulated panel selections, we show that game theory strategies perform significantly better than other veto strategies commonly used in real-world selection games such as courtroom jury selection.

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Introduction

Selection games comprise a class of games where players compete over the composition of a panel of one or more members. Each player is given a limited number of vetoes, or strikes, whereby unfavorable panel members may be stricken and thereby replaced by a member of a replacement pool. Selection games commonly arise in the courtroom jury selection, the selection of academic committee members, corporate board members, debate moderators, the hiring of job applicants, and selection of arbitration judges.

The choice by a rational player to exercise a veto is based on whether doing so will most likely result in a favorable empanelment. However, the outcome from exercising a veto is not always obvious. For example, in some selection systems, players are faced with a decisions to exercise vetoes, having little knowledge of the favorability of potential replacements. This can occur because players lack information regarding the favorability of replacement panelists who have not yet been examined, or because replacement panelists are drawn at random from a pool. Vetoing a seemingly unfavorable panelist may therefore result in the seating of an even less favorable replacement and, at the same time, preclude the player from vetoing other unfavorable panelists due to exhaustion of vetoes. As a consequence, experienced selection game players, such as litigators and trial consultants, recommend factoring a variety of criteria into the decision to veto, including the panelist's favorability, the likelihood of a veto by an opposing player, the favorability of a replacement panelist, and the possibility that exercising a veto would move an unfavorable panelists closer to inclusion in the panel (Hornbrook and Leibold 2008).

Given the complexities involved, an optimal veto strategy may not be apparent or may not seem practical to implement in real-world selection games. As such, the choice of whether to exercise a veto is often left to gut feeling, intuition, rule of thumb, or informed guesswork. For example, in courtroom jury selection, litigants often make significant investments in order to achieve a high degree of certainty regarding the favorability of potential jurors, but have no framework or best practice for the use of this hard-won information in the exercise of juror vetoes (known as peremptory challenges). A need therefore exists for a practical analytical framework to assist players in selection games. Such a framework will help determine, at each decision point of the empanelment process, whether or not exercising a veto will most likely to lead to a favorable empanelment. We present an optimal, game theory based selection framework, and we show that players who adopt such game theory-based selection strategies can gain a significant advantage in the favorability of the selected panel.

Background and Previous Work

Much of the past work on selection games under veto applied to courtroom jury selection. Jury selection is likely the most common application of such games and it often occurs in high stakes cases where the outcome of the process is critical to the litigants, such as capital cases or in high value corporate litigation. Typically, in courtroom jury selection, a panel of potential jurors is provided with written questionnaires to elicit initial information about their qualifications, attitudes, potential prejudices, and general demographic characteristics. A set of these jurors is seated and examined in further detail through verbal questioning by the judge and litigating parties. The jurors are then considered for vetoes by the litigating parties. When all jurors are acceptable to all parties, or when parties have exhausted all of their vetoes, the selection process is complete and the jury is empaneled.

Federal Rule of Criminal Procedure 24(b) and Federal Rule of Civil Procedure 47(b) specify the number of vetoes allotted per party, but do not address the system by which they may be exercised. Federal judges in the United States retain broad discretion over the details of the selection system used for a particular trial, including how and in what order vetoes can be exercised (Kearse and Kearse 1986) (Bermant 1982). Federal judges commonly adopt some form of either the “struck” or the “strike and replace” (a.k.a. “Jury Box”) systems (Kearse and Kearse 1986) (Bermant 1982). These systems differ by the order in which potential jurors are examined (known as voir dire) and vetoes are exercised. The Struck system allows for complete examination of the entire jury panel prior to the exercise of vetoes. The Strike and Replace system allows for examination of replacement jurors only after they have been selected for possible placement on the jury due to the exercise of a previous veto. While statutes and procedures governing jury selection in state courts vary widely, many if not most, courts use some form of Strike and Replace system. For example, a study of criminal trials in 8 superior courts in California found all of them to be using Strike and Replace (Hannaford-Agor and Waters 2004).

The first well-known application of so-called Scientific Jury Selection (SJS) is attributed to the defense in the 1972 Harrisburg Seven Trial of a group of anti-war protesters (Lieberman and Sales 2007). The government chose a highly conservative trial venue which was likely to produce a jury predisposed to convict. The defense, with the help of a group of social scientists, conducted pre-trial research on the local population and, based on the information obtained, identified potentially unfavorable jurors for striking. A hung jury resulted in the release of the defendants. Since that time, SJS has been applied in almost every major litigation (Strier 1999).

Shortly after the application of SJS in Harrisburg trial, theoretical investigations of the application of game theory to jury selection were undertaken (Brams and Davis 1976) (Roth, Kadane, and DeGroot 1977) (Brams and Davis 1978) (DeGroot and Kadane 1980). However, these works provided little practical guidance for litigators in a courtroom. The conclusions of the most recent works were limited to the advantage, if any, of being the first party to make a challenge decision (DeGroot and Kadane 1980). These previous works were additionally criticized as relying on overly restrictive and unrealistic models of the selection process (Tiplitz 1980). Furthermore, as we show below, useful solutions to real-world game theory jury selection problems require recursive calculations over large numbers of game tree decision points. The computing power necessary to make such complex calculations was not widely available in the 1970s and 1980s and the computer mainframe and punch card programming technology of the time was certainly not practical for use by litigants in a courtroom setting. Perhaps as a result of these issues, early work on game theory in jury selection received little attention among practicing jurists. Here we address the issues that have thus far precluded the widespread application of game theory in jury selection and similar selection games of incomplete information under veto.

The Selection Game

For the purposes of this work, we define the canonical selection game to be similar to the courtroom Strike and Replace jury selection system, according to the following criteria:

- Initial examination of panelists is limited to a subset of seated panelists equal in number to the desired panel size,
- Panelists are considered in order by seat,
- Players alternate in the exercise of a limited number of vetoes,
- Upon veto of a panelist, a replacement is selected from a pool, either in order or at random,
- Replacement panelists are examined upon their selection from the pool, and
- When a panelist has been accepted by all players, a player cannot then perform a 'back strike' and veto that panelist at a later time.
- The selection process continues until either no player desires to exercise a veto or all vetoes have been exhausted,

The results presented below can be modified in a straightforward manner to accommodate variations and/or modifications to this canonical system.

We assume each player to have preferences over panelist characteristics, and possibly preferences over combinations of such characteristics. In addition, each party is assumed to have full knowledge of all available information about the panelists. However, it may be the case that at the time a decision to exercise a veto is made, the characteristics of some panelists are still uncertain. Nevertheless, each party is assumed to have beliefs about the distribution of such uncertain characteristics among the population of panelists. For example, for courtroom jury selection, litigants may have information about particular prospective jurors from questionnaires, voir dire, background studies, observations of behavior, dress, etc. and beliefs about the population of jurors based on the results of demographic surveys, focus groups, mock trials, or prior experience with jury selection in the same venue.

We further assume that each player has knowledge of the preferences over panelist characteristics of the other players. This may be a valid assumption in courtroom jury selection when parties have knowledge of opposing parties' theories of prosecution or defense, have made observations of opposing parties' voir dire examinations including the types of questions presented and the types of jurors focused upon, or have observed patterns in the vetoes previously exercised by opposing parties. Some research suggests that, in practice, opposing parties to a litigation tend to have opposing juror preferences, leading, as we shall see below, to a zero-sum selection game (Brams 2011:148-149) (Brams and Davis 1976). Lacking information to the contrary, it is reasonable to assume as a starting point that this is the case, at least for the panelists with the most extreme characteristic values who would be most likely to incur a veto. (Recent proposals to implement negotiated peremptory challenges in litigation would, in effect, require disclosure of litigants' preferences over jurors (Stevenson 2012) (Morrison 2014) making the selection process more strictly amenable to a game theoretic analysis.)

Each player has a limited number of vetoes. We assume that vetoes are exercised publicly such that each player is aware of the vetoes previously exercised by all other players.

Finally, we assume that all players act rationally and strategically, taking account of other players' optimal choices, and that players do not cooperate in their veto decisions. The resulting selection problem is a multi-player Game Theory problem of *incomplete* and *perfect* information.

Definitions and Terminology

We define the set of *l* players, $\boldsymbol{P}$, as

$$P = \{p_1, p_2, \dots, p_l\} \qquad 1$$

For a two-party courtroom litigation with parties consisting of prosecution and defense (or plaintiff and defendant), we write $\boldsymbol{P} = \{p, d\}$.

Let the set

$$V = \{v_1, v_2, \dots, v_l\} \qquad 2$$

represent the number of vetoes available to each party. For a two-party courtroom litigation we write $\boldsymbol{v} = \{v_p, v_d\}$, where v_p and v_d represent the number of vetoes available to the prosecution and defense, respectively.

We wish to select a panel of size N from a pool of size M. There are initially N prospective panelists seated in the N available panel seats. We require a the number of replacement panelists to be at least as large as the number of panel seats plus challenges, such that: $M \geq N + \sum_{i=0}^{l} v_i$.

We define the state of play, S, of the selection process as

$$S = [p_i, \boldsymbol{V}, \boldsymbol{H}, n]. \tag{3}$$

Here p_i represents the 'controlling party', *i.e.,* the party currently making a choice to exercise a veto, the integer n represents the number of panel seats remaining to be filled, and the history, $\boldsymbol{H} = \{\boldsymbol{h_1}, \boldsymbol{h_2}, \dots \boldsymbol{h_{N-n}}\}$, represents the panelists, $\boldsymbol{h_i}$**,** previously accepted by all players. In any given state, $N - n$ seats have been settled and n seats remain to be decided. When player p_i exercises a veto against the current panelist, that panelist is dismissed, a replacement is seated in their place, and the value of v_i is decremented by one. When player p_i accepts the current panelist, the panelist remains seated and control is passed to the next player. If all players have accepted a panelist, the panelist remains seated, the panel seat under consideration is settled, the panelist is added to the history $\boldsymbol{H}$**,** and n is decremented by 1. We will suppress some or all of these parameters when no ambiguity is introduced.

Players assess the favorability of panelists according to a set of k characteristics. For courtroom jury selection, these characteristics may include age, profession, demeanor, behavior, political leanings, answers to jury questionnaires, voir dire responses, and the like. Characteristics describe what is known about each panelist. Each player is assumed to have preferences over these characteristics. We assume that these characteristics can be described on a numerical scale of real numbers. The k-dimensional vector $\boldsymbol{x_j}$ describes the numeric characteristics of panelist *j*. Let the k-dimensional vector space $\boldsymbol{X}$ describe the possible range of characteristic vectors such that for each panelist *j*, $\boldsymbol{x_j} \in \boldsymbol{X}$. Let the tensor $\overline{\boldsymbol{\chi}} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_M}\} \in \boldsymbol{X} \times \boldsymbol{X} \dots \times \boldsymbol{X}$ represent all information known about all panelists. We assume $\overline{\boldsymbol{\chi}}$ to be known by all players. Let the tensor $\boldsymbol{\chi} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_N}\}$ represent all information known about the subset of N currently seated panelists (some of whom may be accepted by all players and some of whom are still under consideration for veto).

We assume characteristic vectors to be distributed among the pool of panelists according to a distribution $\varphi(\boldsymbol{x})$. Each player may have a different opinion of the form of this distribution. For example, in jury selection, litigating parties may each perform their own mock trials or demographic studies and they may reach differing conclusions about the distribution of characteristics among the jury pool. Let $\varphi_i(\boldsymbol{x})$ denote the characteristic distribution according to player *i*.

Let $U_i(\boldsymbol{\chi})$ be a utility function over characteristics $\boldsymbol{\chi}$ used by player p_i to map a characteristic tensor $\boldsymbol{\chi}$ onto real numbers. Let $\boldsymbol{\chi}'$ and $\boldsymbol{\chi}''$ be any two sets of such characteristic tensors. We require that the utility function obey the relation:

$$\boldsymbol{\chi}' \succcurlyeq \boldsymbol{\chi}'' \Leftrightarrow U_i(\boldsymbol{\chi}') \geq U_i(\boldsymbol{\chi}'') \qquad 4$$

where the operator $\succcurlyeq$ means 'prefers or is indifferent between' the two operands. In other words, we require the numerical utility to be greater for panels whose characteristics are preferred by a player and to be equal when the party is indifferent between two panels

Referring to the panelist currently under consideration as the *current panelist* and player currently making a decision whether to veto the current as the *current player*, we define the following regions of $k \, x \, M$-dimensional characteristic parameter space:

$\boldsymbol{R}_*$: The region where the current player prefers to eliminate the current panelist, assuming the opposing players would accept the current panelist.

$\boldsymbol{R}^*$: The region where at least one opposing player prefers to eliminate the current panelist.

$\boldsymbol{R}_a$: The region where no player finds it favorable to eliminate the current panelist.

Optimal Strategy for Two Player Selection Games

We now specialize to the case of two player selection games. We assume a state where a certain number of panel seats have been settled, and the players are considering a particular panelist for the seat currently under consideration. The characteristics of replacement panelists, should any be called, as well as the panelists who will be seated in subsequent seats, are unknown. However, we assume each player to have beliefs about the distribution of characteristics among the panelists in the replacement pool.

The three possible outcomes for the current panelist are A: Accept by both players; B: veto by player 1; and C: accept by player 1 and veto by player 2. This process is summarized by the extensive form game tree shown in Figure 1. Letting $U_i(S)$ be the utility of the state S to the player i, the strategic form of the same sub-game is given by Table 1. The four optimal strategy choices for this sub-game are enumerated in Table 2, the case chosen depending on the relative values to each player of the states A, B and C.

We make the following comments about these cases: 1) When player 1 prefers state B to either state C or state A, knowledge of the opposing player's utility is unnecessary, and 2) The condition $U_1(B) > U_1(C)$ implies that player 1 would prefer to exercise a veto against the current panelist to arrive at state B than to allow player 2 to veto the same panelist and arrive at state C. However, for a Strike and Replace system, starting from a state $S = [(p_2, \{v_1, v_2\}, H, n)]$, states B and C are given by: $B =$

$[p_2, \{v_1 - 1, v_2\}, H, n]$ and $C = [p_2, \{v_1, v_2 - 1\}, H, n]$. Therefore, contrary to what may seem to be common sense, player 1 prefers to arrive at a state of selecting a panelist from a replacement pool with one *fewer* available veto than it would have, had it accepted the original panelist and allowed the player 2 to veto. Following DeGroot and Kandane (DeGroot and Kadane 1980), we term this seemingly counter-intuitive case *irregular.*

Table 3 identifies the optimal actions taken by both players for the regular and irregular cases,

In the regular case, player 1 will veto when it prefers a replacement to the current panelist, unless it knows that player 2 will also veto. In the irregular case, player 1 will veto when it prefers a replacement to the current panelist, and, in addition it will veto preemptively when it knows that player 2 would veto.

Separable Utility Functions – Panelist Ratings

The utility functions, $U_i(\boldsymbol{\chi})$, describes a player's favorability over panels with characteristics $\boldsymbol{\chi} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}\}$, and $\boldsymbol{x_i}$ describing the set of characteristics of the *i*th panelist. These utility functions can be complex functions of panelist characteristic vectors describing, for example, interactions among panelists or the order in which panelists are seated. It is useful to place some restrictions on the form on the utility functions in order to investigate in more detail the behaviors of optimal selection strategies. In the examples below, we assume that utility functions are symmetric and separable, such that:

$$U_i(\boldsymbol{\chi}) = u_i(\boldsymbol{x_1})\, u_i(\boldsymbol{x_2}) \ldots u_i(\boldsymbol{x_N}) \qquad 5$$

We shall call the function $r_{ij} = u_i(x_j)$ the *rating* of panelist *j* by player *i.* Symmetry of the utility function is intuitively compelling since switching any two panelists should not alter the value of the panel. The assumption of separability requires that the value of a panelist can be considered independently of other panelists seated on the panel. This may not be the case in some real-world selection scenarios such as in courtroom jury selection, since, for example, jurors are expected to interact and influence each other during deliberations. Despite this caveat, it is apparently common practice among litigators in jury selection to assign a favorability rating to each juror and to compare these juror ratings when exercising vetoes, thereby neglecting the possibility of interaction. We feel that using individual panelist ratings is a reasonable starting point for many selection games, however, care should be taken when generalizing rating-based approaches, since important information may not be properly modeled. Without loss of generality, we assume ratings to be made on a scale of real numbers from 0 to 10.

Let the distribution $P_i(r)$ denote the probability for obtaining a rating r for party i. We then obtain the following relation between this rating distribution and the conditional distribution over characteristics:

$$P_i(r) = \int_{\boldsymbol{X}} \varphi_i(\boldsymbol{x}|u_i(\boldsymbol{x}) = r)\, d\boldsymbol{x} \qquad 6$$

Here the integral is taken over the k-dimensional space of characteristics, $\boldsymbol{X}$. $P_i(r)$ describes player i's belief in the distribution of ratings among the pool of panelists.

Opposing Interests – Zero Sum Game

In the general case, ratings, or more fundamentally, utilities over panelist characteristics, may be determined independently by each player. Likewise each player may have independent beliefs about the distributions of characteristics among the panelists. As a result, the definition of utility over characteristics covers a broad range of selection problems, including those where multiple players have aligned, partially aligned, or opposing interests. In the following, we shall investigate a class of selection problems in which all players have either directly aligned or directly opposing interests. Such utility functions are subject to the following constraint for all pairs of players, i and j:

$$\text{for all } \chi, \qquad a\, U_i(\chi) + bU_j(\chi) = c \tag{7}$$

Here, a, b, and c are constants. Since utility functions are defined up to an arbitrary positive linear transformation, without loss of generality we can rewrite this constraint as: $U_i(\chi) = \pm U_j(\chi)$. The positive sign is taken when the interests of players i and j are aligned and the negative is taken when the interests of the players are opposed. We further adopt the convention that all players adopt the same utility function, $U(\chi)$, however, with one set of players wishing to maximize the utility and the opposing set of players wishing to minimize the utility.

When panelist ratings are used (as opposed to sets of characteristics), this convention reduces to $r_{ik} = r_{jk}$, for all panelists k and all pairs of players, i,j. In other words, all players assign the same ratings to panelists, however with some players favoring panelists with high ratings and some favoring panelists with low ratings. Two opposing players with utilities subject to this constraint comprise a two-player, zero sum game.

Two-Player, Single Seat Selection Game

We consider a panel comprised of a single seat to be chosen using Strike and Replace selection system by two directly opposing players, each having at least one available veto. Such a scenario may be applicable to situations such as choosing an arbitrator during an arbitration proceeding, filling a job opening by a selection committee, or selecting a corporate officer among a panel of executives. We note that an assumption of independence of ratings among panelists is irrelevant since only one seat is to be filled. Furthermore, theorem 3 of DeGroot and Kadane (DeGroot and Kadane 1980) insures regularity. Without loss of generality, we define panelist ratings such that player 1 wishes to maximize, and player 2 wishes to minimize, the value of the seated panel.

When player 1 moves first, the expected value of the panel to player i (i.e., the expected value of the root node of the game tree), is given by:

$$V_i \;=\; V_i(B)\int_0^{min(r_*,r^*)} P_{\boldsymbol{i}}(r)dr \;+\; \int_{min(r_*,r^*)}^{r^*} rP_{\boldsymbol{i}}(r)dr + V_i(C)\int_{r^*}^{1} P_{\boldsymbol{i}}(r)dr \qquad 8$$

where, for convenience, we have defined the following threshold rating values:

$$\begin{aligned} \boldsymbol{r}_* &= V_p(B) \\ \boldsymbol{r}^* &= V_d(C) \end{aligned} \qquad 9$$

The first term in Equation (8) represents the case where the player 1 elects to veto, the second term represents both players accepting, and the third term represents player 2 electing to veto. Since players may choose their own rating probability distributions, there are no further constraints on the values of r_* and r^*. The integration limit $min(r_*, r^*)$ accounts for cases where both players would veto a panelist and the player 1 would prefer that player 2 exercise the veto.

When the player 2 moves first, the expected panel value is:

$$V_i \;=\; V_i(C)\int_0^{r_*} P_{\boldsymbol{i}}(r)dr \;+\; \int_{r_*}^{max(r_*,r^*)} rP_{\boldsymbol{i}}(r)dr + V_i(B)\int_{max(r_*,r^*)}^{1} P_{\boldsymbol{i}}(r)dr \qquad 10$$

with

$$\boldsymbol{r}_* = V_p(C)$$

$$\boldsymbol{r}^* = V_d(B)$$

Equations (8) and (10) represent game tree root node values calculated recursively in terms of the values of child nodes. Recursion terminates at leaf nodes, i.e., when all players have accepted a panelist, or when a player has exhausted its vetoes, allowing the opposing player to make a unilateral decision between the panelist under examination and a replacement. The expected value of such a unilateral decision is obtained by setting $r^* = 1$ in Equation (8), and by setting $r_* = 0$ in Equation (10).

Example 1

We consider a two-player, single-seat selection game. Two players are selecting an arbitrator, each player having a single veto. Vetoes are to be exercised sequentially with player 1 making the first decision. Arbitrator j_1 is initially seated, with j_2 and j_3 comprising a panel of alternate arbitrators to be drawn at random. The favorability of each arbitrator is measured by a probability of finding in favor of player 1, on which both parties agree. This favorability is described by a numeric rating, $r \in [0,1]$. Based upon prior research, both parties find panelist favorability to be uniformly distributed among the pool of arbitrators. Prior to examination therefor, each arbitrator's expected probability of finding for player 1 is r = 0.5. The complete game tree, $G_{1,}$ is shown in Figure 2 with each leaf node representing a possible seated arbitrators. Since there is only a single seat, the value of the panel is equal to the rating of the arbitrator who is ultimately seated. Player 1 wishes to maximize this value while the player 2 wishes to minimize this value.

We begin by evaluating leaf nodes. Player 2 controls Node 3 with player 1 having exhausted its veto. The value of Node 3 is found from Equation (10) with $r_* = 0$ and $r^* = 0.5$, to be:

$$V_3 = \int_0^{0.5} r\, dr + 0.5 \times \int_{0.5}^{1} dr = 3/8. \qquad 12$$

If, upon examination, J_1 is found to have a rating below 3/8, player 1 will challenge J_1 at node 1, otherwise player 1 will accept J_1.

Player 1 controls Node 4 with the player 2 having exhausted its veto. The value of Node 4 taking $r_* = 0.5$ and $r^* = 1$, is:

$$V_4 = 0.5 \times \int_0^{0.5} dr + \int_{0.5}^{1} r\, dr = 5/8. \qquad 13$$

If upon examination, J_1 is found to have a rating higher than 5/8, player 2 will veto J_1 at node 2, otherwise, player 2 will accept J_1.

If J_1 has a rating between $r_* = 3/8$ and $r_* = 5/8$, both players will accept J_1. Using Equation (8), the expected value of Node 1, i.e. the expected value of the panel prior to examination of J_1, is:

$$V_1 = \frac{3}{8} \times \int_0^{3/8} dr + \int_{3/8}^{5/8} r\, dr + \frac{5}{8} \times \int_0^{5/8} dr = 0.5\ . \qquad 14$$

These results are in agreement with Roth, et. al.,(1977).

Two-Player, Multiple-Seat Selection Game

A multiple seat selection games using a sequential process can be described as a sequence of single seat selection games, each seat being considered in order. When each new seat is considered, the number of vetoes available to each player is equal to the number remaining after completion of the selection process of the previous seats. For a panel of N seats, we describe the selection process for the n^{th} seat as a game G_n. Each leaf node of the G_n game tree is identified with a root node for the game tree, $G_{n+1,}$ representing selection of the next seat. The composite game tree terminates at the leaf nodes of the last tree, G_N. We therefore write the complete game tree for the selection of N seats, Γ_N, as:

$$\Gamma_N = G_1 \times G_2 \times \dots \times G_N, \quad 15$$

where the 'direct product' of games, signified by the operator, ×, specifies that each leaf node of game G_n becomes a root node for a new game G_{n+1}, preserving state, for all n<N. Node values are calculated recursively using Equations (8) and (10), starting with the leaf nodes of G_N and working backward, up the child branches to the node in question.

We assume a state where n-1 out of N panelists have been selected and the players are now considering the panelist for the n^{th} seat. Referring again to Figure 1, assuming player 1 is moves first, the expected values of the nodes A, B and C to player *i* are now given by:

$$\begin{aligned} V_i(A(\boldsymbol{y})) &= E_i(\{\boldsymbol{c}, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{n-1,}}\boldsymbol{y}], N-n\}) \\ V_i(B) &= E_i(\{\boldsymbol{c}_*, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{n-1}}], N-n+1\}) \\ V_i(C) &= E_i(\{\boldsymbol{c}^*, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{n-1}}], N-n+1\}) \end{aligned} \quad 16$$

Where $\boldsymbol{c} = (c_1, c_2)$, $\boldsymbol{c}_* = (c_1 - 1, c_2)$, and $\boldsymbol{c}^* = (c_1, c_2 - 1)$. The expected value of a state, $S \in (A, B, C)$, over the characteristics, $\boldsymbol{x}$, of the k unfilled panel seats, is given by

$$E_i(S) = \int U_i(S) P_i(\boldsymbol{x_1} \dots \boldsymbol{x_k}) d^k \boldsymbol{x} \qquad 17$$

Here, $U_i(S)$, is the utility to player i of state S and $P_i(\boldsymbol{x_1} \dots \boldsymbol{x_k})$ represents the joint probability distribution over the characteristics of k unfilled seats according to player i.

The regions R_*, R^*, and R_a are now subsets of the complete characteristic space R defined by:

$$R_* = \{x \in R : V_i(A(x)) < V_i(B)\}$$

$$R^* = \bigcup_{l \neq i} \{x \in R : V_l(A(x)) < V_l(C)\} \qquad 18$$

$$R_a = R - R_* \cup R^*$$

Here it is assumed that player i is to make the first decision. In the special case of separable utility functions, we may write the expected values of each node as:

$$V_i(A, r_{in}) = r_{i1} r_{i2} \dots r_{ik-1} r_{in} E_i(\{\boldsymbol{c}, N - n\})$$

$$V_i(B) = r_{i1} r_{i2} \dots r_{in-1} E_i(\{\boldsymbol{c}_*, N - n + 1\}) \qquad 19$$

$$V_i(C) = r_{i1} r_{i2} \dots r_{in-1} E_i(\{\boldsymbol{c}^*, N - n + 1\})$$

Here, $E_i(\{\boldsymbol{c}, N - n\})$ is the value to player i of the child tree representing the selection of $N - n$ panelists given the vector $\boldsymbol{c}$ of available challenges starting with the seat, $n + 1$. The threshold ratings for the n^{th} seat are given by:

$$r_* = E_i(\{\boldsymbol{c}_*, N-n+1\})/E_i(\{\boldsymbol{c}, N-n\})$$

and $$r^* = E_i(\{\boldsymbol{c}^*, N-n+1\})/E_i(\{\boldsymbol{c}, N-n\}) \qquad 20$$

The expected value to player i of a given node having child states A, B and C is therefore:

$$V_i = V_i(B)\int_0^{min(r_*,r^*)} P_{\boldsymbol{i}}(r)dr + \int_{min(r_*,r^*)}^{r^*} V_i(A,r)P_{\boldsymbol{i}}(r)dr + V_i(C)\int_{r^*}^{1} P_{\boldsymbol{i}}(r)dr \qquad 21$$

Example 2

A courtroom jury of 2 members is to be selected using the Strike and Replace system. The prosecution and defense each have a single veto. Both parties have identical juror ratings, r, which describe each juror's probability of conviction. Both parties find the jury pool to have a uniform distribution over this characteristic. The jury utility function is given by the product of juror ratings, $U = \prod_{i=1}^{2} r_i$. The prosecution is the first to optionally exercise a veto. The complete game, tree is shown in Figure 3 where jurors J_1 and J_2 are seated in the jury box and replacement jurors R_1 and R_2 comprise the replacement jurors.

We now determine under what conditions the prosecution should veto the first juror, J_1. Using the results of Example 1, we find that the expected value of the second jury seat at node 5 is $V_5 = 0.5$. The expected value for the second jury seat at node 10 is $V_{10} = 0.375$. The expected value for the second seat at node 9 is $V_9 = 0.625$. The defense veto threshold for node 3 is therefore given by

Equations (20) as $r^* = 0.25/0.375 = 0.667$. The prosecution threshold at node 4 is $r_* = 0.25/0.625 = 0.4$. The expected value of node 3 is:

$$V_3 = 0.375 \times \int_0^{0.667} r\,dr + 0.25 \times \int_{0.667}^{1} dr = 0.167 \qquad 22$$

and the expected value of node 4 is:

$$V_4 = 0.625 \times \int_{0.4}^{1} r\,dr + 0.25 \times \int_0^{0.4} dr = 0.167 \qquad 23$$

The game tree is thereby reduced to the tree shown in Figure 4.

Again, applying Equations (20), we find the threshold values r_* = 0.167/0.5 = 0.333, and r^* = 0.363/0.5 = 0.725. The expected value of the jury prior to examination of any jurors is given by:

$$V_1 = 0.167 \times \int_0^{0.333} dr + 0.5 \times \int_{0.333}^{0.725} r\,dr + 0.363 \times \int_{0.725}^{1} dr = 0.259 \qquad 24$$

Numerical Calculation of Veto Thresholds

The number of decision points present in many real-world selection scenarios may preclude manual calculations such as those given in the examples above. Selection of large panels, such as courtroom juries, with each party having even a handful of vetoes, require analysis of game trees comprising hundreds of thousands, or even millions, of decision nodes. Computer algorithms have been developed by the author to evaluate such scenarios. This section presents the results of computer-based numerical studies representing scenarios including up to 12 panel seats and each player having up to 6 vetoes. The results presented assume a Strike and Replace selection system, separable utility functions, and uniform rating pool distributions over a rating scale of 0 – 10. Care should be exercised when attempting to extrapolate the results given here to scenarios using different selection systems, different panelist utility functions, and/or different replacement pool distributions.

Results are reported as variation of veto thresholds, r_* and r^*, as a function of state parameters. Figure 5 shows the variation of veto thresholds with the number of vetoes available to the players, keeping the panel size fixed at a single seat. As expected, the greater the number of vetoes available to player 1, the larger the value of the veto threshold, indicating that player 1 is willing to accept greater risk in challenging a panelist when possessing more vetoes. Likewise, when player 2 has a greater number of available vetoes (lower curves in Figure 5), player 1 must be more conservative in the exercise of each veto, thereby lowering its veto threshold. We note that the veto threshold varies over a significant portion of the rating range, in this case from approximately 2 to 8, for the parameter space sampled (up to 6 vetoes per player). This indicates that game theory solutions to selection processes may differ significantly from intuitive or 'rule of thumb' approaches such as using a fixed veto threshold set at the average value of the replacement pool rating distribution.

Varying Panel Size

Players participating in a Strike and Replace or similar selection system might be expected to be more conservative in the exercise of vetoes when the number of seats remaining to fill is large. All else being equal, a player with a single seat to fill would be expected to set a veto threshold at a higher value than would the same player working to fill multiple seats. This is simply the result of spreading a fixed number of vetoes over a greater number of seats. While the tendency to conserve vetoes may be qualitatively intuitive, the degree to which optimal veto thresholds are sensitive to varying panel size are only calculable within a mathematical framework such as the game theory models described here.

The game theory model for a Strike and Replace selection system with a uniform panelist rating distribution on a scale of 0 to 10 and with both players having a single available veto is used to calculate veto threshold as a function of the number of unfilled panel seats shown in

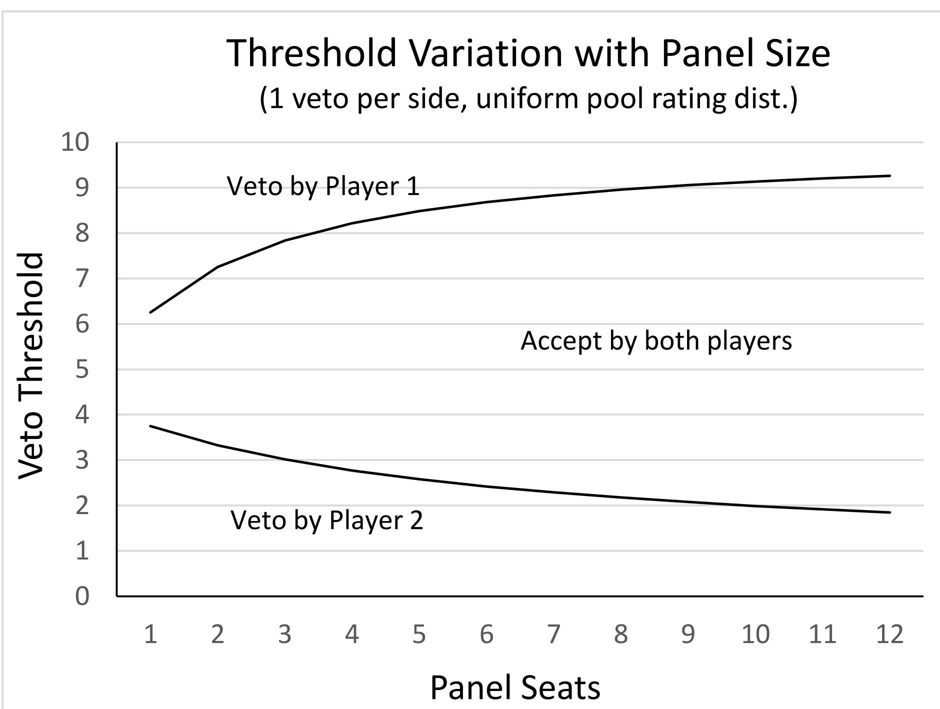

Figure 6. This figure is divided into three regions by the threshold curves: 1) the panelist will be vetoed by the player 2, 2) both players will accept the panelist, and 3) the panelist will be vetoes by player 1. The region of acceptance becomes larger as the panel size increases, reflecting the risk aversion of the players when required to exercise a fixed number of vetoes over a larger number of seats. We note the increasing departure of the game theory-based thresholds from the pool average value as the panel size increases. For a panel of twelve seats, player 2's threshold is over 4 points higher than the pool average of 5.

Challenge Scenarios and Strategies

The relative performance of the game theory-based solutions presented here are determined by 'playing' such solutions against other possible strategies in computerized simulation studies. Due to the probabilistic nature of Strike and Replace panel selection, no single instance of a selection game can determine the relative performance of a strategy. However, over large numbers of plays, meaningful results can be obtained. We define the following strategies for comparison studies:

Game Theory:	The strategy presented in this work
Pool Average:	Veto above (below) the pool average for player 1 (player 2)
Coin Toss:	Veto at random, independent of panelist rating.
Always accept:	Never exercise a veto

Computer simulations of Strike and Replace scenarios have been implemented for various panel sizes and various numbers of vetoes. Results for up to 12 panel seats with each side having 4 available vetoes are shown in Figure 7. Here, player 2 has been programmed to use Game Theory strategy whereas player 1 variously uses Game Theory, Pool Average, Coin Toss, or Always Accept. The resulting panel values are the product of the individual panelist ratings, normalized to a scale of 0 – 10. On this scale, players 1 and 2 implementing equally effective selection strategies would result in a panel rating of 5. Each data point represents 1000 computer-simulated plays.

As expected, the difference in performance between the various strategies becomes larger as the panel size increases, reflecting the greater uncertainties associated with larger panels. Coin Toss and Always Accept, neither of which make use of rating information, perform poorly against both Pool Average and Game Theory over the entire range. Pool Average is able to eliminate the least favorable panelists by using a fixed veto threshold equal to the average rating among the replacement pool, thereby increasing the favorability, for player 1, of the seated panel relative to Coin Toss or Always Accept. Game Theory, however, makes use of a dynamic veto threshold which accounts for changing state including potential replacement panelist ratings, likely actions of opposing players, numbers of remaining vetoes and number of panel seats remaining to fill, and making use of this information performs considerably better than Pool Average, especially for larger panel sizes.

Conclusions

Game theory based selection strategies are designed to maximize the probability of obtaining a favorable empanelment through the strategic use of vetoes during panel selection. We have shown that given a set of selection rules, a set panelist ratings, and a replacement pool rating distribution, the optimal strategic use of vetoes is mathematically calculable through recursive subgame optimization. Two party zero-sum selection games of arbitrary panel size have been analyzed in detail in the case of separable utility functions, with emphasis placed on obtaining veto threshold values as a function of state of play.

To validate our selection game models and optimization solutions, we have presented numerical simulations which compare various strategies for the exercise of vetoes. The strategies considered include Always Accept, Coin Toss, Pool Average and Game Theory. Always Accept and Coin Toss make no use of rating information since vetoes are exercised at random or are not exercised at all. Pool Average makes partial use of rating information, and results in expected panel values higher than Coin Toss and Always Accept, however the fixed veto threshold is not able to account for the changing state of play. As a result, Pool Average is prone to seating panelists who may be even less favorable than the ones who were vetoed. The strategic exercise of vetoes using Game Theory provides a dynamic veto threshold that responds to the state of play and, in particular, accounts for likely opposing party actions. Our numerical studies show that for multi-seat selection scenarios, such as courtroom jury selection, the expected improvement when using Game Theory is significant. This suggests that parties making investments in panelist ratings would do well to make similar investments in the strategic exercise of vetoes using Game Theory rather than adopting Pool Average or some other fixed threshold system.

Experienced selection game players such as litigators and trial consultants may knowingly or intuitively apply game theoretic principles to the exercise of vetoes. However, it would be impractical for them to determine precise veto thresholds in the absence of mathematical calculations described here. Pool Average and other such 'rule of thumb' strategies commonly employed by litigators, may have the advantage of being simple to implement in real-world settings, however, such strategies are not expected to perform as well as game theory strategies. Computing power sufficient to perform game theoretic calculations is currently available in portable laptop computers and will likely soon be available in computer tablets and smart phones. We expect that computer-aided game theory-based selection will become commonplace in courtrooms in the near future.

Finally, the simulations presented here are based on the assumption of separable utility functions and numerical panelist ratings. The results do not consider the effects of panelist interactions, for example during jury during deliberations. Future work incorporating such effects in a game theory analysis would be welcome.

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Figures

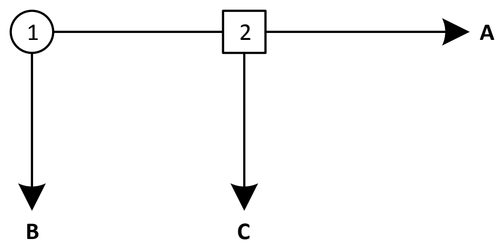

Figure 1. Extensive form game tree representing the decision process for a single panelist. Node 1 represents the decision for player 1. Node 2 represents the decision for player 2. A,B, and C represent the three possible outcome states for this sub-game.

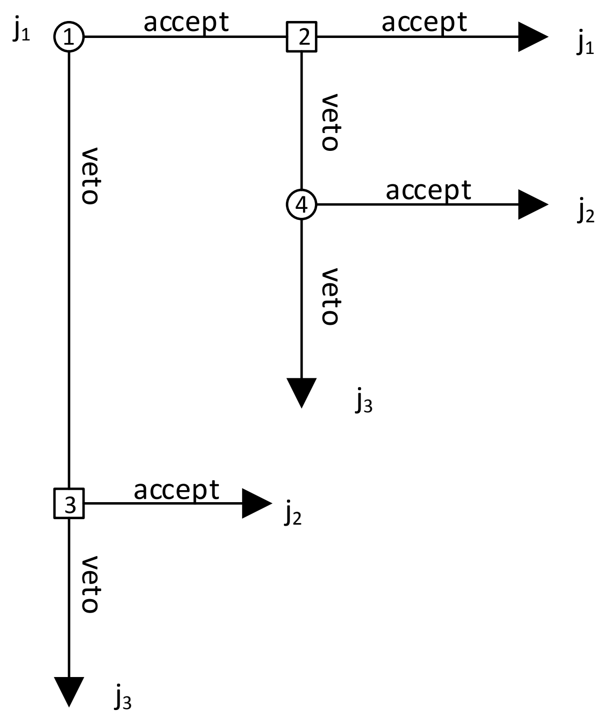

Figure 2. The complete two-player game tree for a single seat panel with each player having a single veto. Circles represent decision points for Player 1. Squares represent decision points for Player 2.

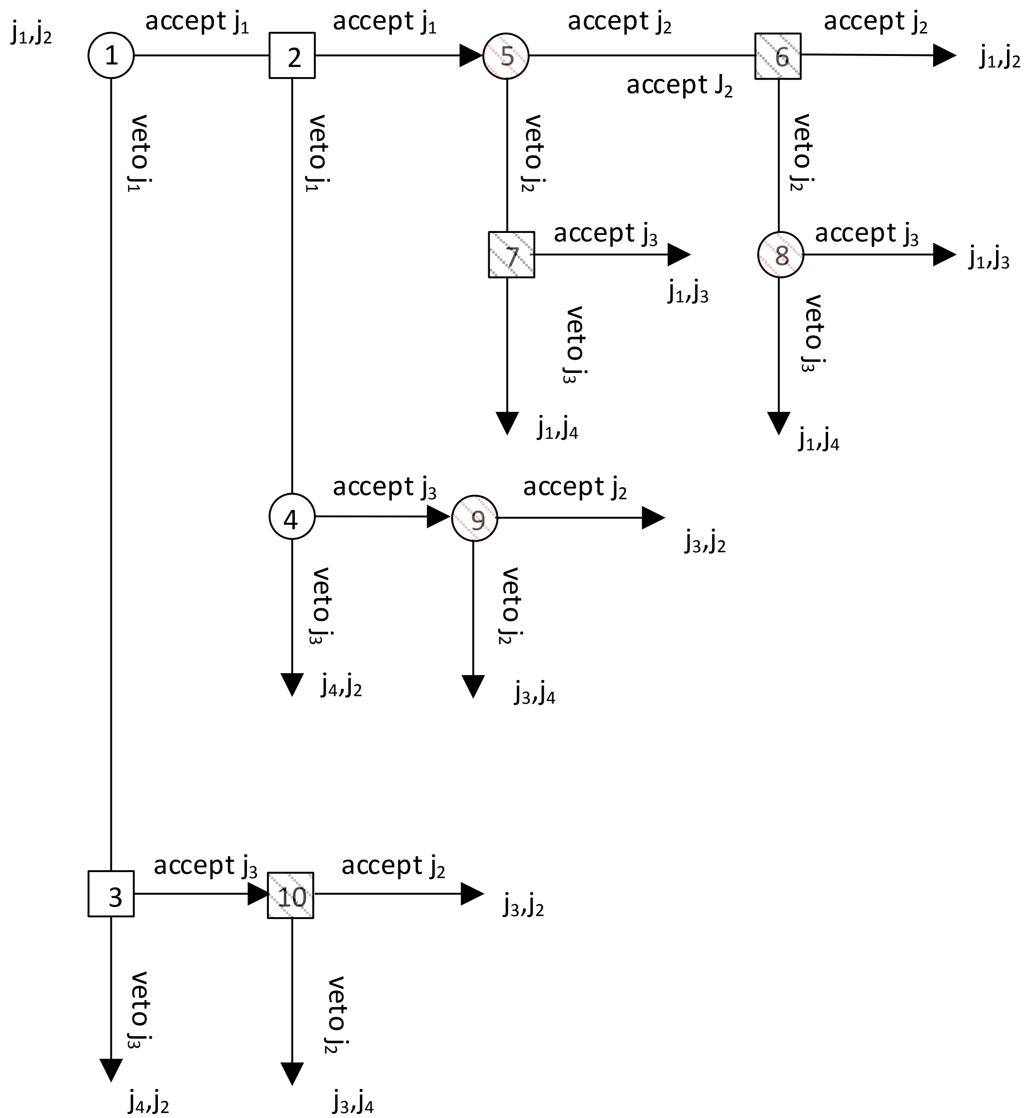

Figure 3. The complete game tree for the selection process described in Example 2. Circles are decision points for Player 1. Squares are decision points for Player 2. Open decision points represent decisions for seat 1. Shaded decision points represent decisions for seat 2. Leaf nodes are marked with possible panel outcomes.

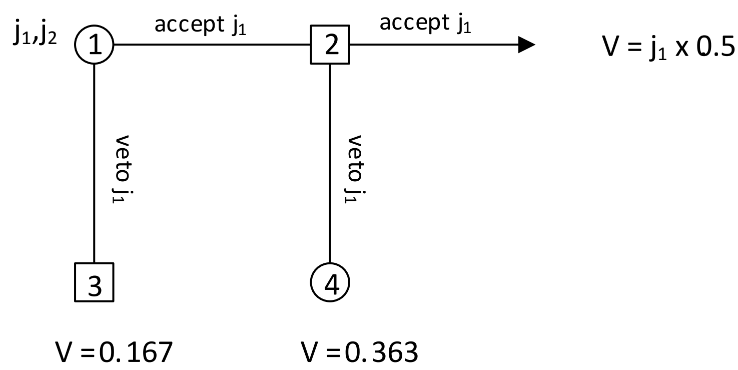

Figure 4. The game tree of Figure 3 reduced by recursive application of Equations (21) and (10).

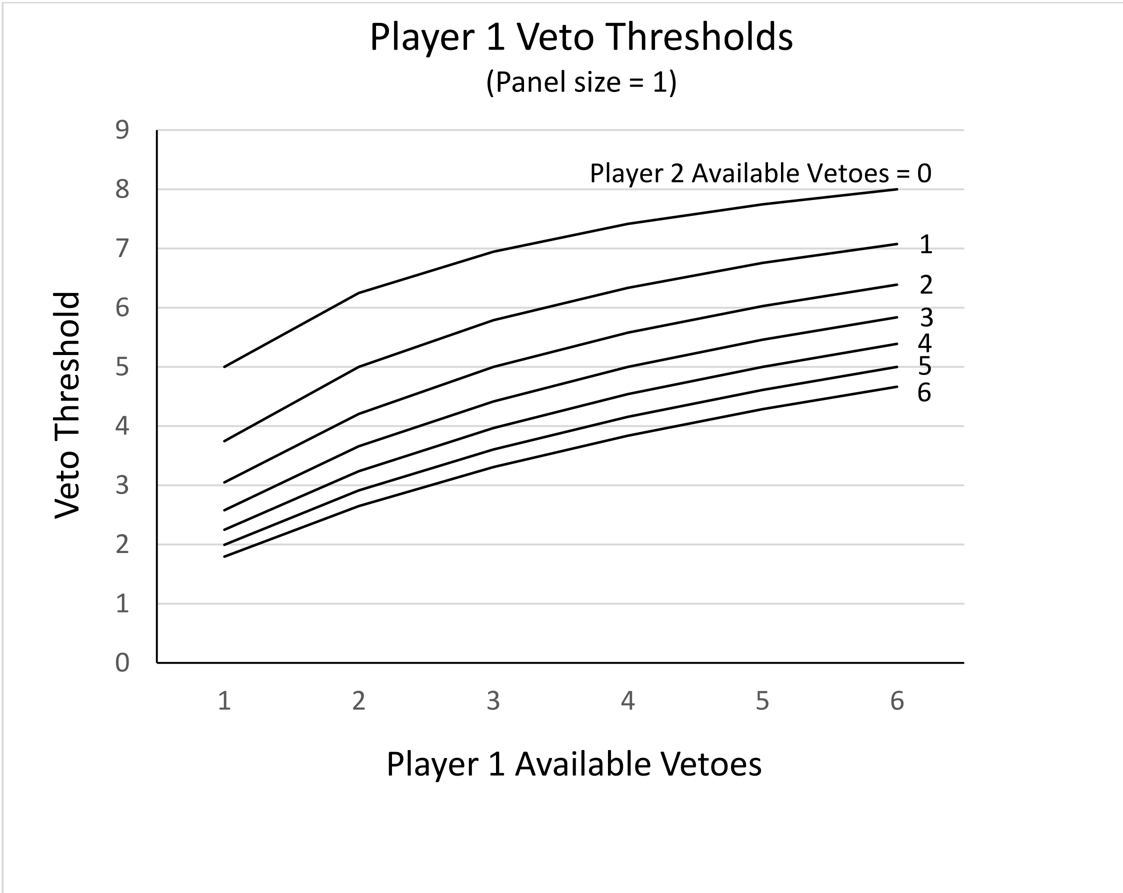

Figure 5. Numerically calculated prosecution challenge threshold values for a single seat jury. Different curves represent different numbers of peremptory challenges available to the defense.

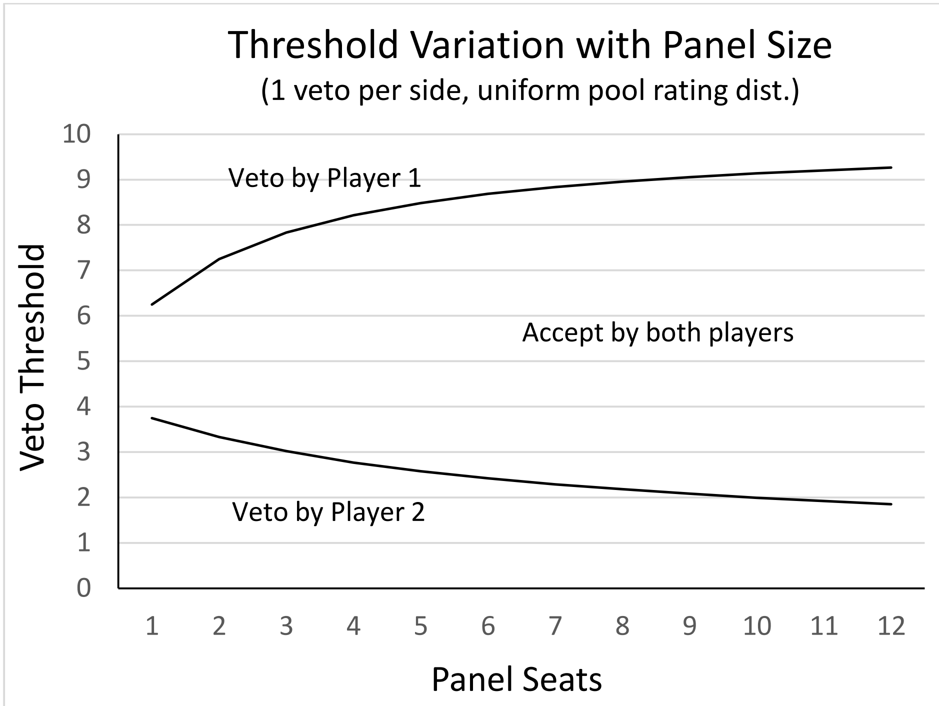

Figure 6. Challenge thresholds for multiple seat juries with each side having a single peremptory challenge.

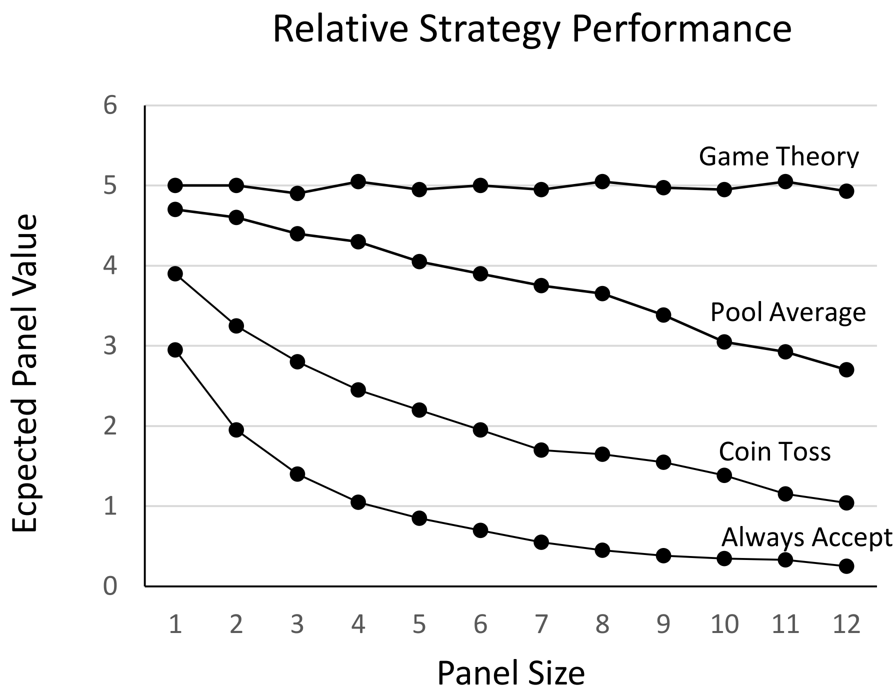

Figure 7. Results of simulation studies of Game Theory played against various strategies for various panel sizes. Player 2 always uses Game Theory. Player 1 uses Game Theory, Pool Average, Coin Toss, or Always Accept. Each side has 4 available vetoes. Since Player 1 wishes to maximize the panel value, Game Theory is the best strategy choice.

Tables

Player 2

Player 2		Accept	Veto
	Accept	$U_1(A),U_2(A)$	$U_1(C),U_2(C)$
	Veto	$U_1(B),U_2(B)$	---

Table 1. Strategic form two-party selection game for a single panelist as in Figure 1.

Case	Conditions	Player 1 Strategy	Player 2 Strategy	Result
1)	$U_1(A) \geq U_1(B)$ $U_2(A) \geq U_2(C)$	Accept	Accept	A
2)	$U_1(C) \geq U_1(B)$ $U_2(C) > U_2(A)$	Accept	Veto	C
3)	$U_1(B) > U_1(A)$ $U_2(A) \geq U_2(C)$	Veto		B
4)	$U_1(B) > U_1(C)$ $U_2(C) > U_2(A)$	Veto		B

Table 2. Optimal strategies for the sub-game shown in Table 1.

depending on the characteristics, x, of the current panelist:

Regular Case:

Condition	Player 1 strategy	Player 2 strategy
$x \in R^*$	Accept	Veto
$x \in R_a$	Accept	Accept
$x \in R_* - R_* \cap R^*$	Veto	------

Irregular Case:

Condition	Player 1 strategy	Player 2 strategy
$x \in R_a$	Accept	Accept
$x \in R_* \cup R^*$	Veto	------

Table 3. Optimal strategies for regular and irregular cases.