SELECTION UNDER VETO WITH LIMITED FORESIGHT

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Abstract

I consider a class of two-party, zero sum sequential selection games where players compete over the composition of a panel comprising one or more seats. The players have a limited number of vetoes which can be used to reject panelists, with replacements being selected at random from a pool. One player wishes to maximize, and the opposing player wishes to minimize a utility function over panelist ratings. This selection game is notable in that 1) it is common, with application in fields such as trial jury selection, selection of job applicants by committee, selection of panelists and moderators for a debate, arbitrator selection, and group decision making, and 2) it leads to complex games, obscuring optimal strategies and thereby increasing the costs of optimal play. In this paper, I consider various simplified, limited foresight panel selection strategies and compare these with optimal play. I find that a commonly used limited foresight strategy – vetoing panelists less favorable than some fixed value, such as the replacement pool average - is not generally effective for selection games, such as jury selection, which comprise more than a few slots.

1. Introduction

Games of selection, wherein two or more parties compete over the composition of a panel, jury, board, or similar body, are commonplace. In the United States, trials by jury, for example, comprise in excess of 150,000 cases per year (U. S. Department of

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State, 2009), each one requiring a jury selection process that can last from minutes to weeks. Similar veto-based selection games include the selection of moderators and panelists for debates (Farah, 2004), selection of an arbitrator (de Clippel, Eliaz, & Knight, 2014), filling job vacancies by committee, and narrowing the field of alternatives in group decision analysis (Justice & Jamieson, 1998). While the details of such games may differ, they typically allow players to veto particular candidate panelists, with the number of allotted vetoes being limited. United States federal criminal trials, for example, require a panel of twelve jurors with prosecution and defense each having six vetoes (known as peremptory challenges) (Federal Rules of Criminal Procedure, Title VI , Rule 14.).

When vetoes are exercised, replacement candidates may be drawn at random from a pool. Because the favorability of replacements may initially be unknown, players are faced with a degree of uncertainty when exercising vetoes. The number of slots in the panel together with the limited number of vetoes and uncertainty in the value of replacements make such selection games particularly complex. Like chess and other complex games, it is not often the case that optimal strategies are known or even calculable by the players. Furthermore, it may be the case that the costs of obtaining optimal solutions, when factored in to the game outcomes, outweigh their benefits. Similar to what has been shown in voting games with financial incentives (Bassi & Williams, 2014), it is possible that only in high value cases will players attempt the deeper strategic analysis required for optimal play.

For these reasons, selection games are prime candidates for the application of so-called 'limited foresight' strategies (Rubenstein, 1998). Limited foresight (LF) strategies attempt to overcome costs associated with the solution of complex games by considering outcomes incrementally, accounting for consequences only within a few 'moves' of a current decision point, and disregarding further consequences as unforeseeable. This work explores the efficacy of LF strategies in two-party, zero sum sequential selection games. The work is organized as follows: In Section 2, we define the 'canonical' selection game which is similar to the so-called Strike and Replace system used in a majority of jury trials in the United States and many other countries. In Section 3, we provide the optimal solution to this

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selection game, and in Section 4 we discuss the complexity of this solution. In Section 5, we provide the lowest order LF approximations. In Section 0, we compare these LF solutions with the optimal solution using the results of Monte Carlo simulations for the particular case of selection games with four vetoes per side. We provide discussion of our results in Section 7. Our main result is that while low order LF solutions can be good strategies under particular conditions, for example, when players have an equal number of remaining vetoes, they are, in general, significantly inferior to optimal play.

2. The Selection Game

A set of N ratings, $R = \{r_1, r_2, \dots, r_N\}$, is chosen, with each rating $r_i \in [0,1]$. Such ratings may represent, for example, the favorability of a panelist. Players P_1 and P_2 consider each rating sequentially and alternate in applying actions A ϵ {*accept*,*veto*}. If a player chooses *accept*, the rating remains in its slot. If a player chooses *veto*, the rating is replaced with a new rating, selected at random from a probability distribution, $\rho(r)$, which we will call the 'pool'. If both players choose *accept* for the rating in slot *i*, then the rating for that slot is settled, and the game passes to slot *i+1*. Players can choose *accept* any number of times throughout the game, however, the number of times *veto* can be chosen is limited to C_1 and C_2, for players P_1 and P_2, respectively. We define the vector C = (C_1, C_2) to describe these veto allotments. P_1 wishes to maximize, and P_2 wishes to minimize, the utility, *U*, given by the formula,

$$U(R) = \prod_i r_i \qquad \text{eq. 1}$$

The game is defined as $G(N, C, \rho)$. In the following, we shall suppress the dependence on the replacement pool rating distribution, ρ.

We define the lower case quantities $\boldsymbol{c} = (c_1, c_2)$ to represent the number of challenges remaining, and the lower case n to represent the number of slots remaining to fill, at a given decision point. The state of play at any given decision point can then be described as $S = (n, \boldsymbol{c})$.

We define the sub-game representing a single slot i as $G_i(c)$. The complete selection game, $G(N, C)$, can be considered as a 'direct product' of sub-games, each such sub-game representing a single slot:

$$G(N, C) = G_1(C) \times G_2(c) \times \ldots \times G_N(c) \qquad \text{eq. 2}$$

The direct product of games is interpreted as follows: Each terminal node of a sub-game G_i is associated with a root node for a subsequent sub-game, G_{i+1}, the state being preserved. For example, if a terminal history for sub-game G_1 results in the use of *a* vetoes for P_1 and *b* vetoes for P_2, then the subsequent sub-game resulting from that history will be the game $G_2(C_1\text{-a}, C_2\text{-b})$, and so on.

A history for any branch in a sub-game terminates under the following conditions:

1. Both players accept a rating.
2. One player accepts and the opposing player has no remaining vetoes.
3. Neither player has a remaining veto.

An example sub-tree for a state with $\boldsymbol{c} = (1,1)$ is shown in Figure 1.

3. Optimal Solution

The game $G(N, C)$ is solvable by backward induction, starting from the terminal nodes of the N^{th} sub-game (Caditz, 2015). Recursion relations to determine the value $V(n, c_1, c_2)$ of each node are as follows:

$$V(n, c_1, c_2) = V(n-1, c_1, c_2)\ \psi\big(r_*(n, c_1 - 1, c_2), r^*(n, c_1, c_2 - 1)\big)$$

$$r_*(n, c_1, c_2) = V(n, c_1 - 1, c_2)\ /\ V(n-1, c_1, c_2) \qquad \text{eq. 3}$$

$$r^*(n, c_1, c_2) = V(n, c_1, c_2 - 1)\ /\ V(n-1, c_1, c_2)$$

Where we have defined the function:

$$\psi(r_*, r^*) \equiv r_* \int_0^{r_*} \rho(r)dr + \int_{r_*}^{r^*} r\rho(r)dr + r^* \int_{r^*}^{1} \rho(r)dr \qquad \text{eq. 4}$$

(We note that zero sum games where both parties agree on the distribution $\rho(r)$ are 'regular', such that in all cases, $r_* \leq r^*$. For a discussion of regularity see (DeGroot & Kandane, 1979), (Caditz, 2015) and references therein. Equation 3 is subject to the boundary conditions:

$$\begin{aligned} r^*(n,0,1) &= \bar{r} \\ r_*(n,0,1) &= 0 \\ r_*(n,1,0) &= \bar{r} \\ r^*(n,1,0) &= 1 \\ V(n,0,0) &= \bar{r}^n \\ V(0,c_1,c_2) &= 1 \end{aligned} \qquad \text{eq. 5}$$

The value $\bar{r}$ is the average over the rating distribution, $\rho(r)$. The quantities r_* and r^* are interpreted as the veto thresholds for the maximizer, P_1, and the minimizer, P_2, respectively. At a given state, $S = (n, \boldsymbol{c})$, a rating, r, such that $r < r_*(s)$ should be vetoed by P_1 and rating, r, such that $r > r^*(s)$ should be vetoed by P_2. Sub-game perfect equilibria (SPE) can therefore be characterized as a set of 'veto thresholds', $\left(r_*(S), r^*(S)\right)$ defined for every decision state, S, and recursively calculable using eq. 3.

4. Game Tree Complexity

We now investigate the complexity of the game tree associated with the sub-game, G_i. Any decision state with $c_1, c_2 \geq 1$, representing the process of selecting a given rating, leads to at least one sub-tree as shown in Figure 2. The number of decision points, $\mathcal{N}(c_1, c_2)$, contained in the sub-tree starting with (c_1,c_2) can therefore be determined recursively as:

$$\mathcal{N}(c_1, c_2) = 2 + \mathcal{N}(c_1 - 1, c_2) + \mathcal{N}(c_1, c_2 - 1), \qquad \text{eq. 6}$$

with the boundary conditions: $\mathcal{N}(m, 0) = \mathcal{N}(0, m) = m.$ Table 1 provides sub-game complexity for several veto allocations. Multiple slot game trees are correspondingly more complex, since all partitions of c_1 and c_2 among the slots must be considered. For example, the game $G(2,1,1)$ is composed of the five sub-games $G_1(1,1)$, $G_2(1,1)$, $G_2(1,0)$, $G_2(0,1)$, and $G_2(0,0)$, and produces ten decision points total.

Optimal play requires solving Equations 3 through 5 for each decision point. This may be a difficult task in real-world situations such as during jury selection in a courtroom. Computer algorithms could, in principle, accomplish optimal play, however, information would have to be updated as panelists are vetoed and replacement panelists are selected from the pool, interviewed and rated. This new rating information would be entered and optimal solutions recalculated throughout the election process. This in itself may impose costs on the players, for example, by distracting them from the ongoing proceedings, or by requiring the services of experts dedicated to performing such calculations. Many practitioners therefore seek reasonable estimates to optimal play which are thought to provide outcomes which, while not optimal, are in some sense acceptable. In the following, we discuss several common strategy choices and compare their expected performance against optimal play.

5. Limited Foresight Strategies

Even when players are aware of the existence of an optimal solution given by eq. 3, applying the necessary recursive calculations to games with large numbers of slots and vetoes may itself be prohibitive. Even in the case of a single slot with two vetoes per side, optimal solution would require 18 such recursions. Real-world selections such as trial juries comprising 12 slots with four or more vetoes per side are exponentially more complex. For example, numerical calculations by the author show that the game $G(12,4,4)$ contains 8,293,348 separate decision points. It would be a daunting task to solve such a case in the absence of computer assistance. In practice, some form of limited foresight (LF) strategy is applied to such cases.

Formal LF strategies are defined by the level of play which a player considers at each decision point. For example, in a *'k = 1'* LF strategy, a player would only look one move ahead. In a slight variation, we define LF models for selection games based on the number of vetoes and slots considered by the player. A player applying LF(n,c_1,c_2) behaves as though they are considering *n* slots with the players have c_1 and c_2 available vetoes, respectively. We feel that this would more closely model the process used by LF decision makers in selection games since they would be focused on the more tangible number of remaining vetoes rather than the more abstract concept of level of play. In the following, we discuss several, lower order LF strategies.

LF(1,0,0): 'Always Accept'

This model contains no decision points. A player behaves as though it has no vetoes and that its opponent has no vetoes. A party implementing LF(1,0,0) will always accept. While it may seem to be an unreasonable strategy, it has been adopted by decision makers who feel that the cost of even understanding the selection problem is too great, or that exercising vetoes may reflect poorly on them, antagonize other decision makers, jurors, panelists, judges, etc. Such externalities are beyond the scope of this work, however we maintain LF(1,0,0) as a viable model for comparison purposes.

LF(1,1,0) and LF(1,0,1): 'Pool Average'

This model contains one decision point. For LF(1,1,0), P behaves as though it has a single veto, to be applied to a single slot. Player P_1 further behaves as though its opponent, P_2, has no vetoes. In this case, P can elect to exercise a veto without considering P_2's potential reaction. Applying, eq. 3, we find $r_* = \bar{r}$. In other words, when exercising a veto, P_1 expects, on average, to obtain a replacement value equal to the pool average. Therefore, if a rating falls below the pool average, P_1 will exercise a veto. If a rating falls above the pool average, P_1 will not veto. The model LF(1,0,1) describes the same condition for player P_2, however with the threshold $r^* = \bar{r}$ reflecting the maximum rating that will be accepted by P_2. Opposing players, each using Pool Average strategies, would never agree on accepting a candidate and would be aggressive in employing vetoes. At least one player, and probably both, will exhaust all vetoes early in the selection process and, having done so would be forced to accept the remaining candidates as they are presented.

LF(1,1,1)

This model contains four decision points. The player assumes that the opposing player will respond to a replacement with LF(1,0,1) or LF(1,1,0), and therefore expects the opponent to exercise a veto, should a replacement rating fall below for P_1, or above for P_2, the pool average, $\bar{r}$. Using eq. 3, we can calculate the threshold values:

$$
\begin{aligned}
r_* &= \int_0^{\bar{r}} r\,\rho(r)dr \;+\; \int_{\bar{r}}^1 \bar{r}\,\rho(r)dr \\
&= \bar{r} - \int_{\bar{r}}^1 (r-\bar{r})\rho(r)dr \\
&\qquad = \bar{r} - \delta_p \qquad\qquad \delta_p \geq 0
\end{aligned}
$$

eq. 7

$$
\begin{aligned}
r^* &= \int_0^{\bar{r}} \bar{r}\,\rho(r)dr \;+\; \int_{\bar{r}}^1 r\,\rho(r)dr \\
&= \bar{r} + \int_0^{\bar{r}} (\bar{r}-r)\rho(r)dr \\
&\qquad = \bar{r} + \delta_q \qquad\qquad \delta_q \geq 0
\end{aligned}
$$

The effect of considering the opposing parties action is to reduce r_* by an amount δ_p, and increase r^* by an amount δ_q, relative to the pool average $\bar{r}$. There is a range of values bracketing the pool average that is acceptable to both players.

Regardless of the limited LF model chosen, we assume that if the terminal node of a history falls within the LF horizon, the player will play the true game rather than the LF model. For example, a player can play LF(1,1,1) unless its opponent has exhausted all vetoes, at which time it would play LF(1,1,0) for P_1, or LF(1,0,1) for P_2.

Example 1

We examine the game $G(6,4,4)$, assuming a uniform pool distribution:

$$\rho(r) = U(1,0) \qquad \text{eq. 8}$$

Numerical calculations show that this game comprises 520,087 decision points contained within 178,751 distinct histories.

Optimal Solution:

As the game proceeds, replacement values are drawn at random as players exercise vetoes. Different repetitions of the game will therefore likely follow different histories. The optimal veto threshold value at any given decision point will depend upon the particular history that is followed. Since replacement ratings are selected at random from the pool, one cannot know which history will be followed in any instance of the game. We therefore examine threshold values over all possible histories using what we will call a 'Leaf Diagram' such as is shown in Figure 3. Figure 3 summarizes all possible optimal threshold values, r_*, that may be obtained by the maximizing player, P_1. The shaded region shows the optimal threshold range as a function of the number of decision points, d, encountered by P_1 in the preceding history, assuming that P_1 was the first to play. Since all histories start at the root node, the first decision point at $d = 1$ has a specific value, in this case, $r_* \approx 0.38$. The longest possible histories, with both players using all available vetoes and P_1 withholding its last veto until the final decision point, in this case with $d = 13$, result in a threshold $r_* = \bar{r} = 0.5$. Between these two extremes, threshold values depend upon the particular history followed. The solid line in Figure 3 represents the average threshold value, $\langle r_* \rangle$, among the histories. Error bars represent one standard deviation of threshold value about the average. The threshold range initially increases with the level of play from the root node value, and then decreases again toward the terminal nodes of the longest histories. The average threshold value increases from the initial $d = 1$ value of $r_* \approx 0.38$ toward the pool average, $r_* = 0.5$, as the level of play increases.

Limited Foresight Models

LF(1,0,0): Since this strategy is equivalent to always accepting, threshold values are given by $r_* = 1$ and $r^* = 0$. With reference to Figure 3, we see that there is no possible history for which $r_* = 1$ is an optimal threshold value. This model can never represent an optimal strategy to the current selection game.

LF(1,1,0): In this case, $r_* = \bar{r} = 0.5$. This value is plotted as the dot-dash line in Figure 3. At early decision points with d < 6, no histories include the threshold $r_* = 0.5$ as an optimal value. Between d = 6 and d = 10, only a small fraction of histories include this value. Only beyond 10 decisions, does the LF(1,1,0) model become consistent with optimal threshold values for a significant fraction of histories. This is simply the result of slots being settled and vetoes being exhausted such that later decision points more closely resemble the LF(1,1,0) model.

LF(1,1,1): In this case, $\bar{r} = 0.5$ and $\delta_p = \delta_q = 0.125$. The threshold values are therefore given by $r_* = 0.375$ and $r^* = 0.625$. Keeping in mind that the last decision in any given history reverts to LF(1,1,0) for P_1 (and LF(1,0,1) for P_2), we calculate the average LF values over all histories. Shorter histories will revert to LF(1,1,0) for P_1 (or LF(1,0,1) for P_2) earlier in the level of play. The average LF values thus calculated for the maximizer, P_1, are shown by the white dotted curve in Figure 3. For the game under consideration, the LF(1,1,1) result is remarkably similar to the average optimal threshold value given by the solid black curve and we would expect LF(1,1,1) to perform well. However, this is not the case in geneal.

Figure 4 shows Leaf Diagrams for a variety of games with differing veto allotments. As in Figure 3, the shaded regions show the numerically calculated optimal threshold values for all possible histories. The solid curve in each graph represents the LF(1,1,1) values averaged over histories. Regions where the solid black curve falls outside the shaded area indicate that LF(1,1,1) model is a poor approximation to the optimal solution. When the maximizer has fewer vetoes than the minimizer, L(1,1,1) overestimates the optimal threshold. When the maximizer has more vetoes than the minimizer, L(1,1,1) underestimates the optimal threshold. As expected, this effect is more pronounced with greater veto imbalances.

6. Performance of Limited Foresight Strategies

To evaluate the performance of LF strategies, we have performed Monte Carlo game simulations, playing various LF strategies against the optimal game theory strategy defined by eq. 2, for the games $G(N,1,1)$, $G(N,4,4)$, $G(N,2,6)$, and $G(N,6,2)$, and for $N = 1$ through $N = 12$. The replacement pool rating distribution is assumed uniform, $\rho(r) = U(0,1)$. Each game was played by computer algorithm and repeated 100,000 times. (These simulations should not be confused with Monte Carlo tree search techniques used in artificial intelligence game models. Such models randomly sample large game trees with known node values to search for favorable states, whereas the current study evaluates entire trees with potentially unknown node values.) Results are reported in Figure 5 as a percentage of the equilibrium game value. We have assumed that the minimizer, P_2, always plays the optimal strategy (P_2 assumes P_1 will play optimally), while the maximizer, P_1, variously plays optimal strategy, LF(1,1,1), LF(1,1,0), and LF(1,0,0). Results are plotted in Figure 5 for these various strategies.

These graphs can be understood as follows: The threshold values for LF(1,1,1) and LF(1,1,0) are approximately 0.375 and 0.5, respectively. Comparison can be made between these values and the optimal threshold values with reference to Figure 4. When optimal threshold values are, on average, close to an LF value, that LF model performs well. This is the case, in general, when P_1 and P_2 have equal numbers of vetoes. When P_1 has fewer vetoes than P_2, optimal threshold values are generally lower than the LF values, thereby reducing the LF performance. When P_1 has more vetoes than P_2, optimal threshold values are, on average, higher than the LF values, resulting, again, in poor performance, however, with somewhat better performance found from L(1,1,0) ($r_* = 0.5$) than from L(1,1,1) ($r_* \approx 0.375$).

7. Discussion

We have presented optimal solutions to common two-player, zero sum selection under veto games, together with limited foresight approximations. The full solution to these games requires somewhat complex recursive calculation of veto thresholds over a potentially large game tree. Common selection games such as those describing trial jury selection may contain up to several million decision points, precluding, for all practical purposes, their solution other than by computer algorithm. We have therefore investigated relatively simple limited foresight models which limit the number of slots and the number of vetoes considered at any given decision point. The simplest models, LF(1,1,0) for the maximizer and LF(1,0,1) for the minimizer, consider only a single slot with the current player having a single veto and the opposing player having none. This amounts to players using the pool average as their veto threshold value. Monte Carlo game simulations show that LF(1,1,0) obtains about 95% of the game value for single slot games (e.g., G(1,4,4)). However, the model fares poorly for more complex multi-slot games (e.g., < 60% of game value for G(12,4,4)) and for games with uneven veto allocations.

The next simplest model, LF(1,1,1), considers a single slot under contention with both players having a single veto. This model produces good results for games with even veto allegations (> 85% of game value for G(12,4,4)), but fares poorly for uneven veto allocations (<75% of game value for G(12,2,6)). The model does better when the party using it has more vetoes than the opposing party (e.g., G(N,6,2) for P_1), however, in such cases, the pool average model (e.g., LF(1,1,0)) may perform even better.

As is the case with any bounded rationality strategy, players employing limited foresight strategies must consider tradeoffs between the costs of obtaining deeper foresight and the benefits afforded by doing so. The LF(1,1,0) and LF(1,0,1) strategies are relatively simple to implement, requiring only the calculation a single pool average, which could be done easily by hand. However, these low order strategies often perform poorly against strategies using deeper foresight, such as LF(1,1,1), under many common conditions. The advantage of using deeper foresight strategies, however, comes at the cost of rapidly increasing complexity. LF(1,1,1), for example, requires the recursive application of eq. 3, including potentially complex integration over the rating probability distribution, over 4 decision points. While this is likely to be much simpler than optimal play which may comprise thousands of decision points, it may nevertheless impose significant calculation costs. In addition, players using LF(1,1,1) and deeper strategies would have to carefully monitor remaining vetoes at each state of play, so that they would know when to switch to lower foresight levels as the game nears a termination or the veto allocations become unbalanced. Finally, the risk of calculation and execution errors may be higher for players using deeper foresight strategies, possibly negating their advantage.

To avoid these risks, it is likely that players using LF(1,1,1) or higher strategies would employ computer assistance for calculating threshold values and tracking vetoes. The costs of employing computer assistance include hardware and software costs, costs associated with learning how to use computer software, as well as subjective costs such as 'giving up control' to computers and distrust of third party products. However, once computing tools are employed, there is little if any additional cost associated with executing the optimal, fully recursive strategy, since integration and recursive calculation, while difficult for humans, are essentially trivial for computers. Once the decision is made to use computing technology, it makes little sense to apply any limited foresight model at all. With this in mind, it would seem that realistic choices are 1) foregoing technology and its associated costs and playing strategies similar to LF(1,1,0) (or LF(1,0,1) for the minimizer) or 2) employing computing technology and playing the full optimal strategy.

A second option is to use a strategy similar to LF(1,1,1), however, with an estimated, rather than calculated, δ value. In other words, P_1 would veto candidates rated lower than δ_1 below pool average and P_2 would veto candidates rated higher than δ_2 above pool average, with δ_1 and δ_2 set by the respective players using intuition or experience. Such 'pseudo' LF play may afford some of the benefits of LF, however, without the associated calculation costs. This, however, is risky, since intuition may not be trusted in complex situations, especially so when veto allotments are uneven and even true LF strategies do not produce good results.

The analysis presented here assumes a common rating system, known to both players, over which players have opposing interests. This amounts to a zero-sum selection game of complete information. In many situations, although players' ultimate interests may be directly opposed, there is no guarantee that they will evaluate ratings in the same manner and adopt identical rating systems. For example, although parties to a jury trial have opposing interests in the outcome of the trial, their attorneys may have different opinions of the favorability of jurors toward their preferred outcome. There is, however, some statistical evidence that jury selection does proceed as a zero-sum game. A recent study investigated over 700 felony trials in two counties in Florida, found that the vetoes exercised by prosecutors and defense were opposing when considering a rating scale based on age (Anwar, Bayer, & Hjalmarsson, 2012). In this study, prosecutors tended to veto younger jurors while defense attorneys tended to veto older jurors.

In a similar manner, the form of the utility function may differ from the multiplicatively separable utility given by eq. 1. This may be the case where there are interactions between candidates, such that the favorability of each depends on the characteristics of the others. Again, to use the example of a jury trial, an easily influenced or gullible juror may seem favorable if there are other favorable seated jurors who have an influential personality type. However, the same gullible juror would seem unfavorable if the influential jurors were unfavorable. In future work we hope to explore how LF models are in fact used by decision makers, and whether our conclusions regarding the efficacy of LF models hold true for non-zero sum games and games subject to more complex utilities.

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Figures

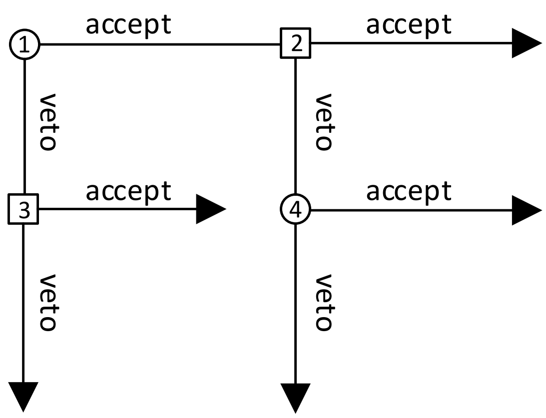

Figure 1. Game sub-tree for P_1 and P_2 each having a single veto. Circles represent decision points for P_1 and squares represent decision points for P_1. Play starts with P_1 at node 1.

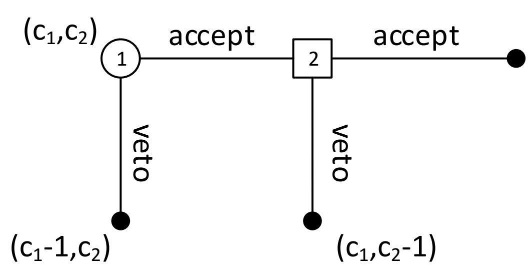

Figure 2. A sub-tree representing the decision process for a single rating (seat or jury member). Players P_1 and P_2 have c_1 and c_2 vetoes, respectively.

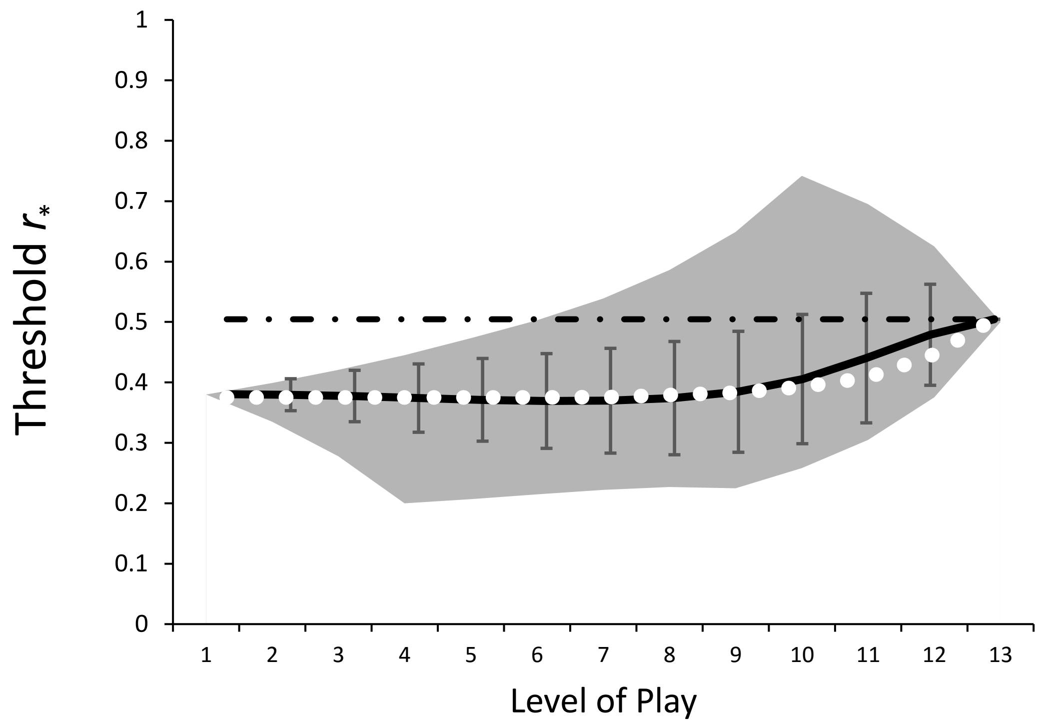

Figure 3. Veto threshold values for player P as a function of level of play for the game $G(6,4,4)$ and a uniform pool distribution, $\rho(r) = U(0,1)$. The shaded area shows the optimal threshold range for all possible histories. The solid line is the average threshold with error bars representing one standard deviation over histories. The dot-dash line shows the L(1,1,0) veto threshold. The white dotted curve shows the average of the L(1,1,1) thresholds over histories.

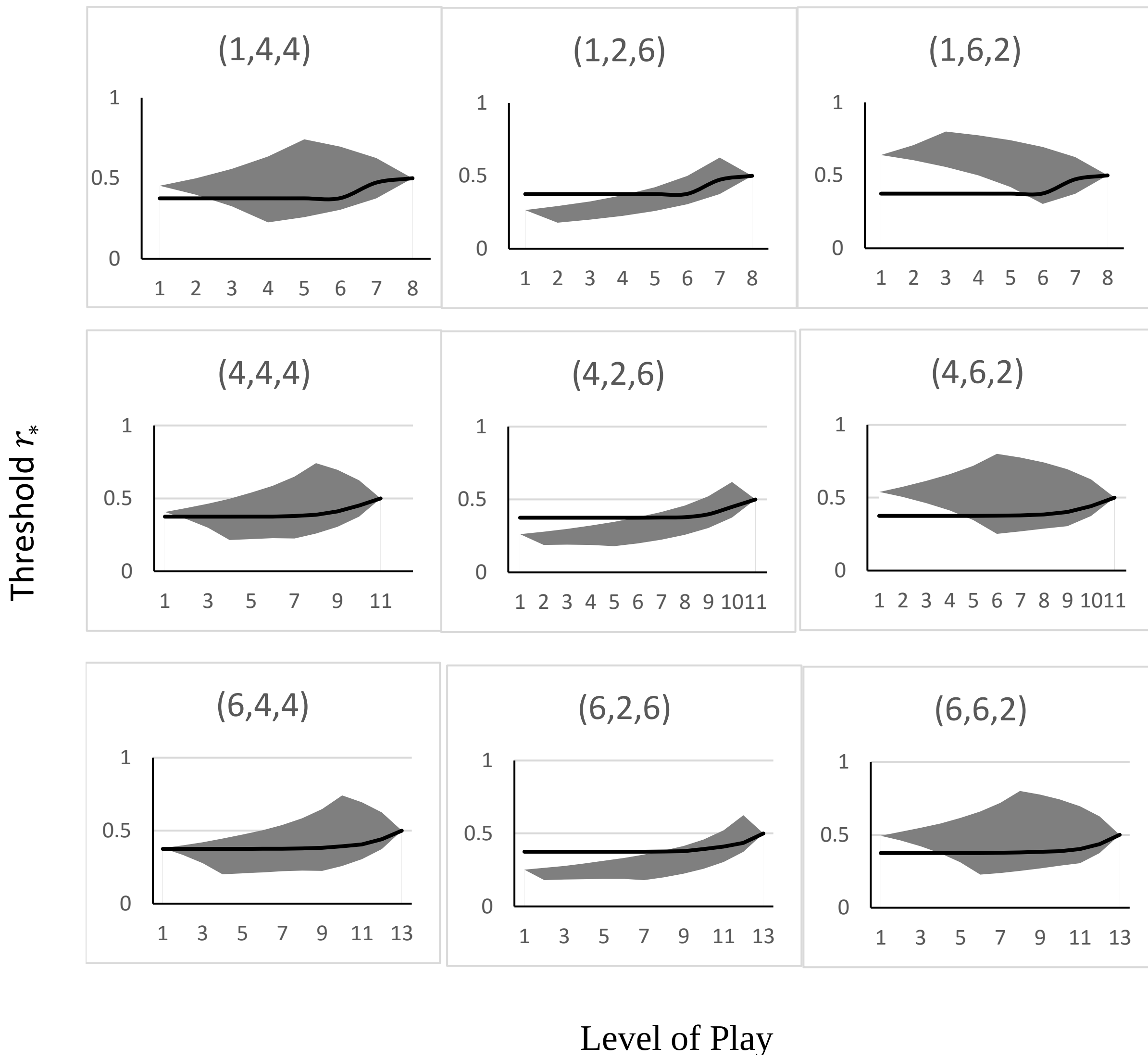

Figure 4. Plots showing player P veto threshold r_* vs. decision level for various selection games. Shaded regions show thresholds over all possible histories. The solid curve shows the L(1,1,1) LF approximation. L(1,1,1) performs poorly for games with a veto imbalance.

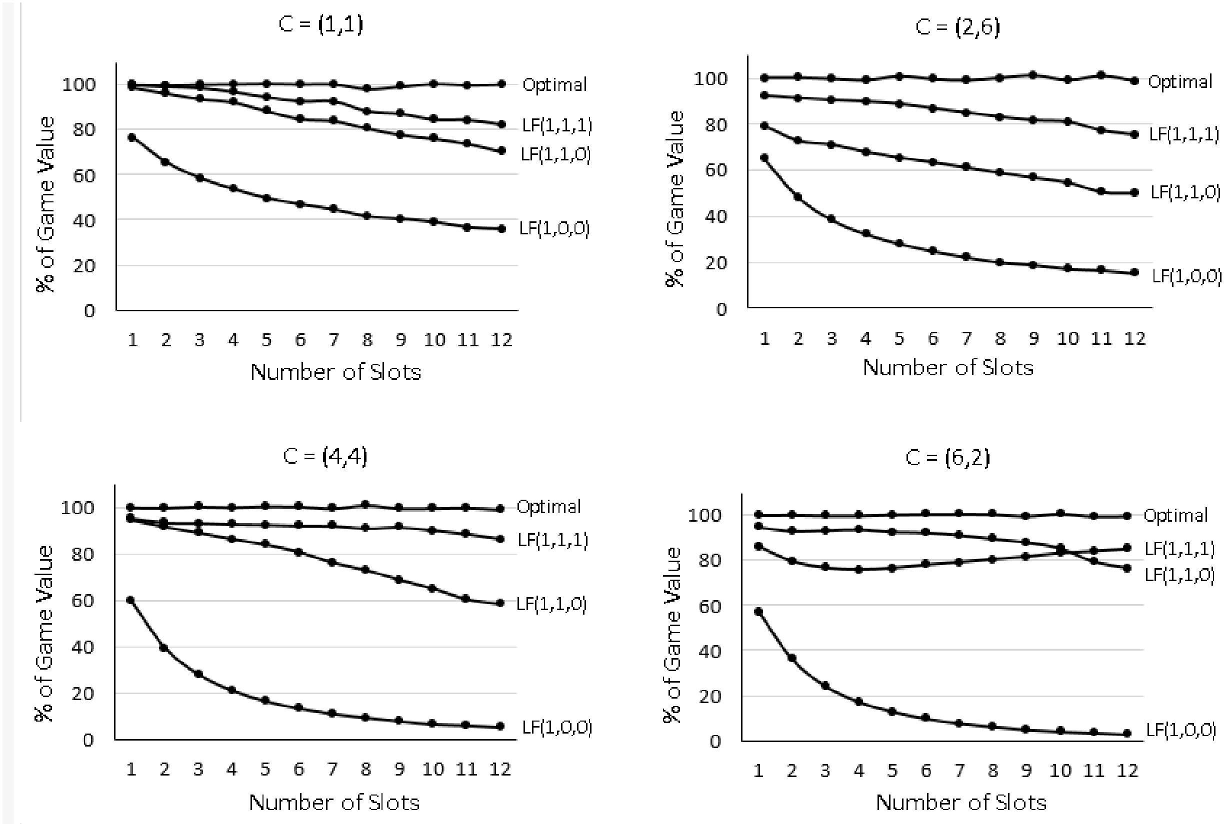

Figure 5. Monte Carlo Game simulations for games with C=(1,1), C=(4,4), C=(2,6) and C=(6,2). Three LF strategies and one optimal strategy are played by the maximizer, P, against the minimizer, Q, who always plays optimally. Game outcomes for the maximizer, P, are plotted as the average of the percent of game value. Each data point represents 100,000 simulated selection games and a uniform rating pool distribution, $\rho(r) = 1$, is assumed.

Tables

		C_1						
		0	**1**	**2**	**3**	**4**	**5**	**6**
	0	0	1	2	3	4	5	6
	1	1	4	8	13	19	26	34
	2	2	8	18	33	54	82	118
C_2	**3**	3	13	33	68	124	208	328
	4	4	19	54	124	250	460	790
	5	5	26	82	208	460	922	1714
	6	6	34	118	328	790	1714	3430

Table 1. Game complexity for various veto allocations for a single seat panel selection game. The values represent the number of decision points contained in the game tree. Game complexity increases rapidly with larger veto allocations.