STRATEGIC JURY SELECTION GAMES – A THEORETICAL ANALYSIS

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Abstract

I consider a class of selection games where members of a group such as a jury, panel or committee, are to be selected from among a larger pool of individuals by two or more parties with non-aligned interests. Each party may exercise a limited number of vetoes, or challenges, against members who are found to be unfavorable. However, at the time of such a challenge, little may be known about potential replacements. I derive game theory based challenge strategies for single-seat juries and for multi-seat juries where jury utilities are separable functions of individual juror utility. I provide numerically derived optimal challenge thresholds and we show how these thresholds vary with jury size and with the number of challenges available to each party. Finally, using computer- simulated jury selections, I show that game theory based strategies perform significantly better than other challenge strategies commonly used in trials by jury.

1. Introduction

This paper pertains to methods for selection of jury members currently in use in court systems in the United States and worldwide. Most such jurisdictions provide litigants with the opportunity to challenge potential jury members, for cause or peremptorily, with the goals of making the empaneled jury impartial, unbiased, and agreeable to all parties, and allowing litigants some amount of

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participation in determining the final makeup of the jury. It is generally recognized that the exercise of such challenges can have a significant impact on the outcome of a trial by jury (Abramson, 1994) (Berg, 2006) (Marder, 2006) with some experts believing that perhaps 85% of case outcomes are determined when the jury is selected (Fahringer, 1993-1994). Parties to high stakes and high profile cases often invest significant amounts of time and expense in the application of so-called 'Scientific Jury Selection" (SJS) techniques to evaluate the likely reactions of potential jurors to their theories of prosecution or defense. Jurors identified as unfavorable may be challenged and thereby stricken from the jury. Recent, high-profile examples of the application of relatively sophisticated jury selection techniques include the 1989 trial of Oliver North in which 22 peremptory challenges were exercised, the 1992 trial of the police officers accused of the beating of Rodney King in which the lone black juror was struck by the defense, the 1994 O.J. Simpson murder trial in which the jury selection process continued for two months with the exercise of 20 peremptory challenges (Linder D. O., 2014), the 2004 Scott Peterson murder trial in which 30 peremptory challenges were exercised (Beratlis, et al., 2007), the 2011 Raj Rajaratnam insider trading trial during which 20 peremptory challenges were exercised in a period of about 20 minutes, and the 2013 George Zimmerman murder trial during which seven peremptory challenges were exercised during the selection of a six jurors plus four alternates. The outcomes of each of these trials is perceived to have been influenced to a large degree by the composition of the empaneled jury (Linder, The Trials of Los Angeles Police Officers' in Connection with the Beating of Rodney King, 2007) (Dobbs, 2007).

The choice by a litigant to exercise a peremptory challenge against a potential juror is ultimately based on whether doing so will result in the final empaneled jury being most favorable to that litigant. The choice is not always

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obvious, even when the favorability or ranking of each individual juror is known. For example, it is often the case that a litigant is faced with a decision to exercise a peremptory challenge against a juror, having little knowledge of the favorability of a potential replacement juror selected at random or in order from a jury pool. Challenging a seemingly unfavorable juror may result in the seating of an even less favorable replacement juror and, at the same time, may preclude a litigant from challenging unfavorable jurors later in the selection process due to exhaustion of challenges. As a consequence, experienced litigators and trial consultants recommend factoring a variety of criteria into the decision to challenge a juror, including the juror's rating, the likelihood of a strike of the juror by the opposing counsel, the favorability of a replacement juror, and the possibility that exercising a strike would move an unfavorable juror closer to inclusion in the panel (Hornbrook & Leibold, 2008).

To make best possible use of peremptory challenges, a litigant must choose a strategy designed to optimize the favorability of the final empaneled jury, taking account of the favorability of each individual juror, the favorability of potential replacement jurors, the characteristics of the jury pool, the process used for juror strikes, and an understanding of the opposing party's challenge strategy. Given the complexities involved, an optimal challenge strategy may not be apparent or practical to determine in a courtroom setting. In spite of the great lengths litigants go to in order to profile and rate potential jurors, the choice of whether to challenge a particular juror is more often than not left to gut feeling, intuition, rule of thumb, or informed guesswork. The current state of the art therefore leaves litigants in the awkward position of having made a significant investment in order to achieve a high degree of certainty regarding the favorability of jurors, but with no framework or best practice as to how to use this hard-won information in the exercise of challenges. A need therefore exists for such a framework to assist a litigant in managing the jury selection process and, in particular, to assist in the selection of a challenge strategy that will help determine at each decision point of the jury empanelment process whether or not challenging a particular juror is most likely to lead to a favorable jury empanelment. We will show that litigants who do adopt such strategies are likely to gain a significant advantage in jury selection.

2. Previous Work

The first application of Scientific Jury Selection (SJS) is attributed to the defense in the 1972 Harrisburg Seven Trial of a group of anti-war protesters (Lieberman & Sales, 2007). The government chose a highly conservative trial venue which was likely to produce a jury predisposed to convict. The defense, with the help of a group of social scientists, conducted pre-trial research on the local population and, based on the information obtained, challenged potential problematic jurors. A hung jury resulted in the release of the defendants. Since that time, SJS has been applied in almost every major litigation (Strier, 1999).

Shortly after the application of SJS in Harrisburg trial, theoretical investigations of the application of game theory to jury selection were undertaken (Brams & Davis, A Game-Theory Approach to Jury Selection, 1976) (Roth, Kandane, & DeGroot, 1977) (Brams & Davis, 1978) (DeGroot & Kandane, 1980). These works provided little practical guidance for litigators in a courtroom. The conclusions of the most recent work were limited to the advantage, if any, of being the first party to make a challenge decision (DeGroot & Kandane, 1980). The previous works were additionally criticized as relying on overly restrictive and unrealistic models of the selection process (Tiplitz, 1980). Furthermore, as we show below, useful solutions to real-world game theory jury selection problems require recursive calculations over large numbers of game tree decision points. The computing power necessary to make such complex calculations was not widely available in the 1970s and 1980s and the computer mainframe and punch card programming technology of the time was certainly not practical for use by litigants in a courtroom setting. Perhaps as a result of these issues, early work on game theory in jury selection received little attention among practicing jurists. We now hope to pick up the baton by addressing some of the issues that have thus far precluded the widespread application of game theory in jury selection.

3. 'Strike and Replace' Jury Selection

Federal court trial judges in the United States retain broad discretion in the selection system used including how and in what order peremptory challenges can be exercised in their courtroom (United States v. Severino, 1986) (Bermant, 1982). Federal Rule of Criminal Procedure 24(b) and Federal Rule of Civil Procedure 47(b) specify the number of peremptory challenges allotted per party, but do not address the system by which challenges may be exercised. Federal trial judges commonly adopt some form of either "struck" or "strike and replace" (a.k.a. "Jury Box") system (United States v. Severino, 1986) (Bermant, 1982). Generally, these systems differ in the order in which voir dire is conducted and challenges are exercised. The struck system allows for complete examination of the entire jury panel prior to the exercise of any challenges. The strike and replace system allows for replacement juror voir dire after they have been seated due to the exercise of a challenge. Statutes and procedures governing jury selection in state courts vary widely with some states. A study of 18 criminal trials in 8 superior courts in California found all courts using some form of strike and replace (Hannaford-Agor & Waters, 2004).

For the purposes of this work, we define the canonical strike and replace jury selection system as one which meets the following criteria:

1) voir dire is initially applied to a subset of cause-free jurors equal in number to the desired jury size,

2) jurors are considered in order by seat,

3) parties alternate in the exercise of a limited number of peremptory challenges,

4) upon challenge of a juror, a replacement juror is selected from a panel, either in order or at random,

5) voir dire is applied to replacement jurors upon their selection, and

6) The selection process continues until either no party desires to exercise a challenge or all challenges have been exhausted.

Furthermore, we assume that once a juror has been accepted by all parties, a party cannot then perform a 'back strike' and challenge that juror at a later time. The results presented below can be modified in a straightforward manner to accommodate variations and/or modifications to this system.

Typically, a panel of potential jurors is provided with a written questionnaire to elicit initial information about their qualifications, attitudes, potential prejudices, and general demographic characteristics. A set of these jurors is seated and examined in further detail through verbal questioning by the Judge and litigating parties. The jurors are then considered seat-by-seat for challenges by the litigating parties. If a juror is challenged, a replacement juror is drawn from the jury panel and seated in their place. This replacement juror is then examined and possibly subject to further challenges. When all litigants accept a juror for a seat, that juror is empaneled on the jury and the process begins anew for the next seat in sequence. When all seats have been accepted, or when the parties have exhausted all peremptory challenges, the jury selection process is complete.

The Strike and Replace method can be further divided into two systems which differ in the way replacement jurors are selected when a juror is challenged. The *unordered* system selects replacement jurors at random from a jury pool, while the *ordered* system selects replacement jurors in order from a jury pool. A litigant exercising a challenge under the unordered system must assume that the rating of a replacement juror will be the expected value of the statistical distribution of ratings among the jury panel. In contrast, the ordered system provides litigants know the identities of replacement jurors before the challenge is exercised, though the replacement jurors may not yet have undergone a detailed examination. Some jurisdictions implement a system which combined ordered and unordered replacement by selecting an ordered subset of the venire as replacement jurors. If this subset is then exhausted due to challenges, a new subset is randomly selected from the venire.

4. Statement of the Problem and Terminology

We consider a case of two or more litigating parties undergoing strike and replace jury selection. We assume that dismissals for cause and hardship have been exercised and that the venire is therefore comprised of qualified, cause-free jurors. Each litigating party has preferences over juror characteristics, and possibly preferences over combinations of such characteristics. In addition, each party is assumed to have full knowledge of all available information about the jurors comprising the venire from questionnaires, voir dire, background studies, observations of behavior, dress, etc. However, it may be the case that at the time a decision to exercise a challenge is made, the characteristics of some jurors are still uncertain. Such a situation may arise when, for example, a challenge is to be exercised before the next replacement juror has undergone voir dire, or when replacement jurors are selected at random from the venire so that a challenging party does not know who the replacement juror will be. However, each party is assumed to have beliefs about the distribution of such characteristics within the venire, possibly based on the results of demographic surveys, prior experience in the same venue, or mock trials.

We further assume that each party has knowledge of the juror preferences the other parties. This may be a valid assumption when parties have knowledge of opposing parties' theories of prosecution or defense, have made observations of opposing parties' voir dire examinations including the types of questions presented and the types of jurors focused upon, or have observed patterns in the challenges previously exercised by opposing parties. Some research suggests that, in practice, opposing parties to a litigation tend to have opposing juror preferences, leading, as we shall see below, to a zero-sum selection game (Brams S. J., 2011, pp. 148-149) (Brams & Davis, 1976). Lacking information to the contrary, it is reasonable to assume as a starting point that this is the case, at least for the most pro-prosecution or pro-defense jurors who would be most likely to incur a challenge. Recent proposals to implement negotiated peremptory challenges would, in effect, require disclosure of litigants' preferences over jurors (Stevenson, 2012) (Morrison, 2014) making the selection process even more amenable to a game theoretic analysis.

Each litigant has a limited number of peremptory challenges to be exercised against jurors whom they feel may be unfavorable, should they be seated in the jury. We assume that challenges are exercised publicly such that each party is aware of the challenges previously exercised by all other parties.

Finally, we assume that all parties act rationally and strategically, taking account of opposing parties' optimal choices, and that parties do not cooperate in their decisions to challenge. The resulting optimization problem is a multi-player Game Theory problem of *incomplete* and *perfect* information.

Let l denote the number of independent parties involved in litigation. We define the set of litigating parties, $\boldsymbol{L}$, as

$$\boldsymbol{L} = \{L_1, L_2, \dots, L_l\}.$$

For a two-party action with parties consisting of prosecution and defense (or plaintiff and defendant), we write $\boldsymbol{L} = (p, d)$.

Let the set $\boldsymbol{C} = \{c_1, c, \dots, c_l\}$ represent the number of challenges available to each party. For a two-party action with parties consisting of prosecution and defense (or plaintiff and defendant), we write $\boldsymbol{C} = (c_p, c_d)$.

We presume a venire of qualified (cause-free) jurors of size M has been selected. Let $\boldsymbol{J} = \{J_1, J_2, \dots, J_N\}$ describe the set of N jurors initially seated in the N available jury box seats. Let $\boldsymbol{K} = \{K_1, K_2, \dots, K_{M-N}\}$ describe the set of M - N available replacement jurors, either seated in order or selected at random from a jury pool. We require a the number of replacement jurors to be at least as large as the number of jury seats plus challenges, such that: $M \geq \ N + \sum_{i=0}^{l} C_i$.

We define the state of play, S, of a given step in the jury selection process as

$$S = \{L_i, \boldsymbol{C}, \boldsymbol{Y}, j\}.$$

Here L_i represents the 'controlling party', i.e., the party currently making a choice to exercise a peremptory challenge. As stated above, $\boldsymbol{C}$ represents the current set of available peremptory challenges, the vector $\boldsymbol{Y}$ represents the jurors previously accepted by all parties, the integer j represents the number of jury seats remaining to be filled. We will suppress some or all of these parameters when no ambiguity is introduced. When party L_i exercises a challenge, the challenged juror is dismissed, a juror from the replacement panel is seated in their place, and the value of c_i is decremented by one. When party i accepts the current juror, the juror remains seated and control is passed to the next party. If all parties have accepted a juror, the juror remains seated, the jury seat under consideration is settled and the juror is added to the vector $\boldsymbol{Y}$. We assume that all seats prior to seat s have been settled and therefore jurors J_1 through J_{s-1} will be on the empanelled jury.

Let there be a set of K characteristics such as age, profession, demeanor, behavior, political leanings, answers to jury questionnaires, voir dire responses, and the like. These characteristics describe what is known about each prospective juror and each party is assumed to have preferences over these characteristics. We assume that each such characteristic can be described on a numerical scale of real numbers. The K-dimensional vector $\boldsymbol{x_j}$ describes the characteristics of juror j. Let the K-dimensional vector space $\boldsymbol{X}$ describe the possible range of juror characteristics such that for each juror j, $\boldsymbol{x_j} \in \boldsymbol{X}$. Let the tensor $\overline{\boldsymbol{\chi}} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_M}\} \in \boldsymbol{X} \times \boldsymbol{X} \dots \times \boldsymbol{X}$ represent all information known about all jurors in the venire. We assume $\overline{\boldsymbol{\chi}}$ to be known by all parties. Let the tensor $\boldsymbol{\chi} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_N}\}$ represent all information known about the subset of N jurors seated in the jury box.

We assume juror characteristics to be distributed among the venire according to a distribution $\varphi(\boldsymbol{x})$. Each party may have a different opinion of the form of this distribution. For example, parties may each perform their own mock trials or demographic studies and reach differing conclusions about the distribution of characteristics among the jury pool. Let $\varphi_i(\boldsymbol{x})$ denote the juror characteristic distribution according to party *i*.

Let $U_i(\boldsymbol{\chi})$ be a utility function over characteristics $\boldsymbol{\chi}$ used by party i to map the characteristics of the seated juror onto real numbers. Let $\boldsymbol{\chi}'$ and $\boldsymbol{\chi}''$ be any two sets of such juror characteristics. We require that the utility function obey the relation:

$$\boldsymbol{\chi}' \succcurlyeq \boldsymbol{\chi}'' \Leftrightarrow U_i(\boldsymbol{\chi}') \geq U_i(\boldsymbol{\chi}'')$$

where the operator $\succcurlyeq$ means 'prefers or is indifferent between' the two operands. In other words, we require the numerical utility to be greater for juries whose characteristics are preferred by a party and to be equal when the party is indifferent between two juries. We assume that each party knows its own utility function. We shall make assumptions below regarding the degree to which each party is aware of the utility functions adopted by other parties.

Referring to the party currently making a decision whether or not to challenge the juror currently under consideration as the *current party*, we define the following regions of $k\,x\,M$-dimensional characteristic parameter space:

1) $\boldsymbol{R}_*$: The region where the currently party prefers to eliminate the current juror, assuming the opposing parties would accept the juror.

2) $\boldsymbol{R}^*$: The region where at least one opposing party prefers to eliminate the current juror.

3) $\boldsymbol{R}_a$: The region where no party finds it favorable to eliminate the current juror.

5. Optimal Strategies for Two Parties

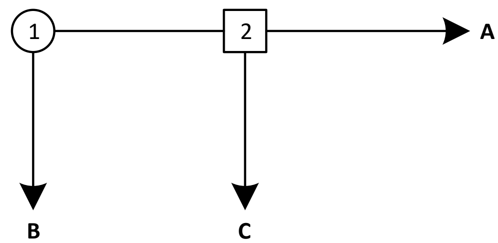

Figure 1. A two-party game tree representing the decision process for a single juror

We now specialize to the case of two party litigation. We assume that a certain number of jury seats have been settled, and the parties are considering a particular juror for the seat currently under consideration. The characteristics of replacement jurors, should any be called, as well as the jurors who will be seated in subsequent seats, are unknown, however, the replacement jurors are drawn from a pool with a distribution of characteristics estimated by each party. The three possible outcomes for the current juror are A: Accept by both parties, B: challenge by Party 1, and C: accept by Party 1 and challenge by Party 2. This process is summarized by the extensive form game tree shown in Figure 1. Letting $U_i(S)$ be the utility of the state S to the party i, the strategic form of the same game is given by Table 1.

		Party 2	
		Accept	Challenge
Party 1	Accept	$U_1(A),U_2(A)$	$U_1(C),U_2(C)$
	Challenge	$U_1(B),U_2(B)$	---

Table 1

The following four optimal strategy choices can be enumerated, the case chosen depending on the relative values to each party of the states A, B and C[1]:

Case	Conditions	Party 1 Strategy	Party 2 Strategy	Result
1)	$U_1(A) \geq U_1(B)$ $U_2(A) \geq U_2(C)$	Accept	Accept	A
2)	$U_1(C) \geq U_1(B)$	Accept	Challenge	C

[1] When party 1 prefers state B to either state C or state A, knowledge of the opposing party's utility is unnecessary.

	$U_2(C) > U_2(A)$		
3)	$U_1(B) > U_1(A)$ $U_2(A) \geq U_2(C)$	Challenge	B
4)	$U_1(B) > U_1(C)$ $U_2(C) > U_2(A)$	Challenge	B

Of these, case 4 requires further discussion. The condition $U_1(B) > U_1(C)$ implies that Party 1 would prefer to exercise a challenge against the current juror to arrive at state B than to allow Party 2 to challenge the juror and arrive at state C. However, for a Strike and Replace system, starting from the state $S = [c_1, c_2, j]$, states B and C are given by:

$$B = [c_1 - 1, c_2, j]$$

$$C = [c_1, c_2 - 1, j]$$

Contrary to what may seem to be common sense, Party 1 prefers to arrive at a state of selecting jurors with one *fewer* challenge than it otherwise would by accepting the juror and allowing the opposing party to challenge. Following DeGroot and Kandane (DeGroot & Kandane, 1980), we term this seemingly counter-intuitive case *irregular*.

We now identify the optimal actions taken by both parties for the regular and irregular cases, depending on the characteristics, *x*, of the current juror:

Regular:

Condition	Party 1 strategy	Party 2 strategy

$x \in R^*$	Accept	Challenge
$x \in R_a$	Accept	Accept
$x \in R_* - R_* \cap R^*$	Challenge	------

Irregular:

Condition	Party 1 strategy	Party 2 strategy
$x \in R_a$	Accept	Accept
$x \in R_* \cup R^*$	Challenge	------

In the regular case, Party 1 will challenge when it prefers a replacement to the current juror, unless it knows that Party 2 will also challenge. In the irregular case, Party 1 will challenge when it prefers a replacement to the current juror, and, in addition it will challenge preemptively when it knows that Party 2 would challenge.

6. Separable Utility Functions – Juror Ratings

In some examples below, we assume that utility functions are symmetric and multiplicatively separable by juror, such that:

$$U_i(\boldsymbol{\chi}) = u_i(\boldsymbol{x_1})\, u_i(\boldsymbol{x_2}) \dots u_i(\boldsymbol{x_N}) \tag{1}$$

We shall call the function $r_{ij} = u_i(\mathbf{x}_j)$ the *rating* of juror j by party i. Symmetry of the utility function is intuitively compelling since switching any two jurors should not alter the value of the jury. The assumption of separability essentially requires that the value of a juror can be considered independently of other jurors seated on the jury. This may not be the case in real-world jury selection scenarios after the first jury pool, since jurors can interact during subsequent deliberations. For instance, the favorability of an easily influenced juror may depend on the favorability of another particularly influential juror seated in the jury. Despite this caveat, it is apparently common practice among litigators to obtain a single favorability rating for each juror and to compare these juror ratings when exercising peremptory challenges. Attempting to exercise challenges based on the totality of underlying juror characteristics and potential interactions may be impractical in a courtroom setting.[2] We feel that obtaining individual juror ratings is a reasonable starting point for jury selection, however, care should be exercised when interpreting rating-based approaches to jury selection, since important information

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[2] This is the crux of the disagreement between those who believe in systematic, mathematical approach to the exercise of peremptory challenges and those who prefer an intuitive approach. For practical reasons, the systematic approach may require the application of a single numeric rating scale or juror ranking. However, experienced jury selectors may intuit that such scales cannot reflect the totality of information available regarding the underlying juror characteristics and any algorithmic result must be taken as a recommendation to be followed in light of the totality of available information.

such as potential juror interactions may not be properly considered. Without loss of generality, we assume ratings to be made on a scale of real numbers from 0 to 10.

Let the distribution $P_i(r)$ denote the probability for obtaining a rating r for party i. We then obtain the following relation between this distribution and the conditional distribution over characteristics:

$$P_i(r) = \int_{\boldsymbol{X}} \varphi_i(\boldsymbol{x}|u_i(\boldsymbol{x}) = r)\, d\boldsymbol{x}$$

where the integral is taken over the K-dimensional space of characteristics, $\boldsymbol{X}$. Let the vector $\boldsymbol{P} = (P_1, P_2, \dots . P_L)$ describe the set of such distributions for the L litigating parties.

7. Opposing Interests

In the general case, juror ratings, or more fundamentally, utilities over juror characteristics may be determined independently by each party. Likewise each party can have independent beliefs about the distributions of characteristics among the venire. As a result, the definition of utility over juror characteristics covers a broad range of jury selection problems, including those where multiple parties may have aligned, partially aligned, or opposing interests. In the following, we shall investigate a class of jury selection problems in which all parties have either directly aligned or directly opposing interests[3]. Such utility functions are subject to the following constraint for all pairs of parties, i and j:

[3] We distinguish between the ultimate objectives of the litigants which may be conviction or acquittal and the favorability of juror characteristics which litigants may interpret independently. Several parties may desire to seat or challenge a particular juror although the party's ultimate objectives may differ.

for all χ, $$a\, U_i(\chi) + bU_j(\chi) = c \qquad (2)$$

Here, a, b, and c are constants. Since utility functions are defined up to an arbitrary positive linear transformation, without loss of generality we can rewrite this constraint as: $U_i(\chi) = \pm U_j(\chi)$. The positive sign is taken when the interests of parties i and j are aligned and the negative is taken when the interests of the parties are opposed. We further adopt the convention, without loss of generality, that all parties adopt the same utility function, $U(\chi)$, however, with one set of parties wishing to maximize and the opposing set of parties wishing to minimize the utility.

When juror ratings are used as described above, this convention reduces to $r_{ik} = r_{jk}$, for all jurors k and all pairs of parties, *i,j*. In other words, all parties assign the same ratings to jurors, however with some parties favoring jurors with high ratings and some favoring jurors with low ratings. Two opposing parties with utilities subject to this constraint comprise a two-party, zero sum game.

8. Two Party Litigation, Single Seat Jury

We consider a jury comprised of a single seat to be chosen using Strike and Replace selection system by two directly opposing parties, each having at least one available challenge. Such a scenario may be applicable to situations such as choosing an arbitrator during an arbitration proceeding, filling a job opening by a selection committee, or selecting a corporate officer among a panel of executives. We note that an assumption of independence of ratings among jurors is irrelevant since only one seat is to be filled. Furthermore, theorem 3 of DeGroot and Kandane (DeGroot & Kandane, 1980) insures regularity. We define the following threshold rating values:

$$\boldsymbol{r}_* = V_p(B)$$
$$\boldsymbol{r}^* = V_d(C) \tag{3}$$

The expected value of the root node to party i is then given by:

$$V_i = V_i(B) \int_0^{min(r_*,r^*)} P_{\boldsymbol{i}}(r)dr + \int_{min(r_*,r^*)}^{r^*} rP_{\boldsymbol{i}}(r)dr + V_i(C) \int_{r^*}^{1} P_{\boldsymbol{i}}(r)dr \tag{4}$$

When the prosecution moves first. When the defense moves first,

$$V_i = V_i(B) \int_0^{r_*} P_{\boldsymbol{i}}(r)dr + \int_{r_*}^{max(r_*,r^*)} rP_{\boldsymbol{i}}(r)dr + V_i(B) \int_{max(r_*,r^*)}^{1} P_{\boldsymbol{i}}(r)dr \tag{5}$$

With states B and C exchanged in the threshold rating definitions (3).

The first term in (4) represents the case where the prosecution elects to challenge, the second term represents both parties accepting, and the third term represents the defense electing to challenge. Since parties may choose their own rating probability distributions, there are no further constraints on the values of r_* and r^*. The integration limit $min(r_*, r^*)$ accounts for cases where both parties would challenge a juror and the prosecution would prefer that the defense challenge. Similar considerations apply to (5)

Equations (4) and (5) provide node values recursively in terms of the values of child nodes. Recursion ends when all parties have accepted a juror, or when a party has exhausted its challenges, allowing the opposing party to make a unilateral decision between the juror under examination and a replacement juror. The expected value of such a unilateral decision is obtained by setting $r^* = 1$ when the prosecution is to make the final decision, and by setting $r_* = 0$ when the defense is to make the final decision.

Example 1

We consider a jury comprised of a single seat. Prosecution and defense each have a single challenge, the prosecution making the first decision. Juror J_1 is seated in the jury box, with jurors J_2 and J_3 comprising a panel of replacement jurors to be drawn at random. The favorability of each juror is measured by a probability of conviction, r, on which both parties agree. Based upon demographic analysis, both parties find the probability of conviction to be uniformly distributed among the venire on a scale of [0,1]. Prior to examination, each juror's expected rating is r = 0.5. The complete game tree, $G_{1,}$ is shown in Figure 2 with a possible resulting jury shown at each leaf node. Since there is only a single jury seat, the value of the resulting jury is equal to the rating of the juror who is ultimately seated. The prosecution wishes to maximize this value while the defense wishes to minimize this value.

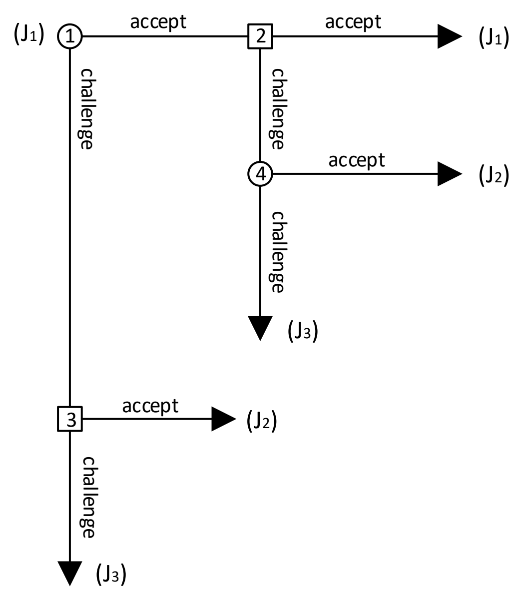

Figure 2. Complete two-party game tree for a single jury seat with each party having a single challenge.

The defense controls Node 3 with the prosecution having exhausted all challenges. The value of Node 3 is found from (4) with $r_* = 0$ and $r^* = 0.5$ to be $V_3 = \int_0^{0.5} r\,dr + 0.5 \times \int_{0.5}^{1} dr = 3/8$. If upon examination, Juror J_1 is found to have a rating below 3/8, the prosecution will challenge J_1 at node 1, otherwise the prosecution will accept J_1.

The prosecution controls Node 4 with the defense having exhausted all challenges. The value of Node 4 taking $r_* = 0.5$ and $r^* = 1$, is $V_4 = 0.5 \times \int_0^{0.5} dr + \int_{0.5}^{1} r\,dr = 5/8$. If upon examination, juror J_1 is found to have a rating higher than 5/8, the defense will challenge J_1 at node 2, otherwise, the defense will accept J_1.

If J_1 has a rating between $r_* = 3/8$ and $r_* = 5/8$, both prosecution and defense will accept J_1. Using (4), the expected value of Node 1, i.e. the expected value of the jury prior to examination of J_1, is:

$$V_1 = \frac{3}{8} \times \int_0^{3/8} dr + \int_{3/8}^{5/8} r\, dr + \frac{5}{8} \times \int_0^{5/8} dr = 0.5 .$$

The above results are in agreement with Roth, et. al.,(1977)

9. Multiple Seat Juries

A multiple seat jury selected using the sequential process can be described as a sequence of single seat selection processes, each seat being considered in order. When each new seat is considered, the number of challenges available to each party are equal to the number remaining after the selection process of the previous seats. For a jury of N seats, we describe the selection process for the i^{th} seat as a game tree G_i. Each leaf node of G_i becomes a root node for the next seat represented by game tree G_{i+1}. The composite game tree terminates at the leaf nodes of the last tree, G_N. We therefore write the complete game tree for the selection of N seats, Γ_N, as:

$$\Gamma_N = G_1 \times G_2 \times \ldots \times G_N,$$

where the ‘direct product’ of games, signified by the operator, $\times$, specifies that each leaf node of game G_i becomes a root node for a new game G_{i+1}, preserving state, for all $i<N$. Node values are calculated recursively using (4), starting with the leaf nodes of G_N and working backward, up the child branches to the node in question.

We assume that k-1 out of N jurors have already been selected and the parties are now selecting the k[th] juror with characteristics $\boldsymbol{y}$ for the seat**,** k. Referring again to Figure 1, the expected values of the nodes A, B and C are:

$$V_i\big(A(\boldsymbol{y})\big) = \ E_i\big(\boldsymbol{c}, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{k-1}}, \boldsymbol{y}], N-k\big)$$

$$V_i(B) = \ E_i(\boldsymbol{c}_*, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{k-1}}], N-k+1)$$

$$V_i(C) = \ E_i(\boldsymbol{c}^*, [\boldsymbol{y_1}, \dots, \boldsymbol{y_{k-1}}], N-k+1)$$

Where

$$\boldsymbol{c} = (c_1, c_2, \dots c_M)$$

$$\boldsymbol{c}_* = (c_1, c_2, \dots, c_i - 1, \dots c_M)$$

$$\boldsymbol{c}^* = (c_1, c_2, \dots, c_l - 1, \dots c_M)$$

and the expected value of a state, S, over the characteristics, $\boldsymbol{x}$, of the unfilled jury seats, is given by

$$E_i(S) = \int \ U_i(S) P_i\big(\boldsymbol{x_1} \dots \boldsymbol{x_j}\big) d^j \boldsymbol{x}$$

Here, $U_i(S)$, is the utility to party i or state S. $P_i\big(\boldsymbol{x_1} \dots \boldsymbol{x_j}\big)$ represents the joint probability distribution for party i over the characteristics of j unfilled jury seats. The definition of $\boldsymbol{c}^*$assumes that some party l has challenged the current juror.

The regions R_*, R^*, and R_a are now subsets of the complete characteristic space R defined by:

$$R_* = \{x \in R \,:\, V_i(A(x)) < \ V_i(B)\}$$

$$R^* = \bigcup_{l \neq i} \{x \in R : V_l(A(x)) < V_l(C)\} \tag{6}$$

$$R_a = R - R_* \cup R^*$$

where it is assumed that party i is to make the first decision. In the case of separable utility functions, we may write the expected values of each node as:

$$V_i(A, r_{ik}) = r_{i1} r_{i2} \ldots r_{ik-1} r_{ik} E_i(\boldsymbol{c}, N-k)$$

$$V_i(B) = r_{i1} r_{i2} \ldots r_{ik-1} E_i(\boldsymbol{c}_*, N-k+1) \tag{7}$$

$$V_i(C) = r_{i1} r_{i2} \ldots r_{ik-1} E_i(\boldsymbol{c}^*, N-k+1)$$

Here, $E_i(\boldsymbol{c}, N-k)$ is the value to party i of the child tree representing the selection of j jurors given the vector $\boldsymbol{c}$ of available challenges for the next jury seat, $k+1$ and we have shown the explicit dependence of the value of node A on r_{ik}.The threshold ratings for the k^{th} seat are now given by:

$$r_* = E_i(\boldsymbol{c}_*, N-k+1)/E_i(\boldsymbol{c}, N-k) \tag{8}$$

and

$$r^* = E_i(\boldsymbol{c}^*, N-k+1)/E_i(\boldsymbol{c}, N-k)$$

The expected value to party i of a node is therefore:

$$V_i = V_i(B)\int_0^{min(r_*,r^*)} P_{\boldsymbol{i}}(r)dr + \int_{min(r_*,r^*)}^{r^*} V_i(A,r)P_{\boldsymbol{i}}(r)dr + V_i(C)\int_{r^*}^{1} P_{\boldsymbol{i}}(r)dr \tag{9}$$

Example 2

A Jury of 2 members is to be selected using the Strike and Replace system. The prosecution and defense each have a single peremptory challenge. Both parties have identical juror ratings, r, which describe each juror's probability of conviction. Both parties find the jury pool to have a uniform distribution over probability of conviction. The jury utility function is given by the product of juror ratings, $U = \prod_{i=1}^{2} r_i$. The prosecution is the first to optionally exercise a challenge. The complete game, tree is shown in **Error! Reference source not found.** where jurors J_1 and J_2 are seated in the jury box and replacement jurors R_1 and R_2 comprise the jury pool.

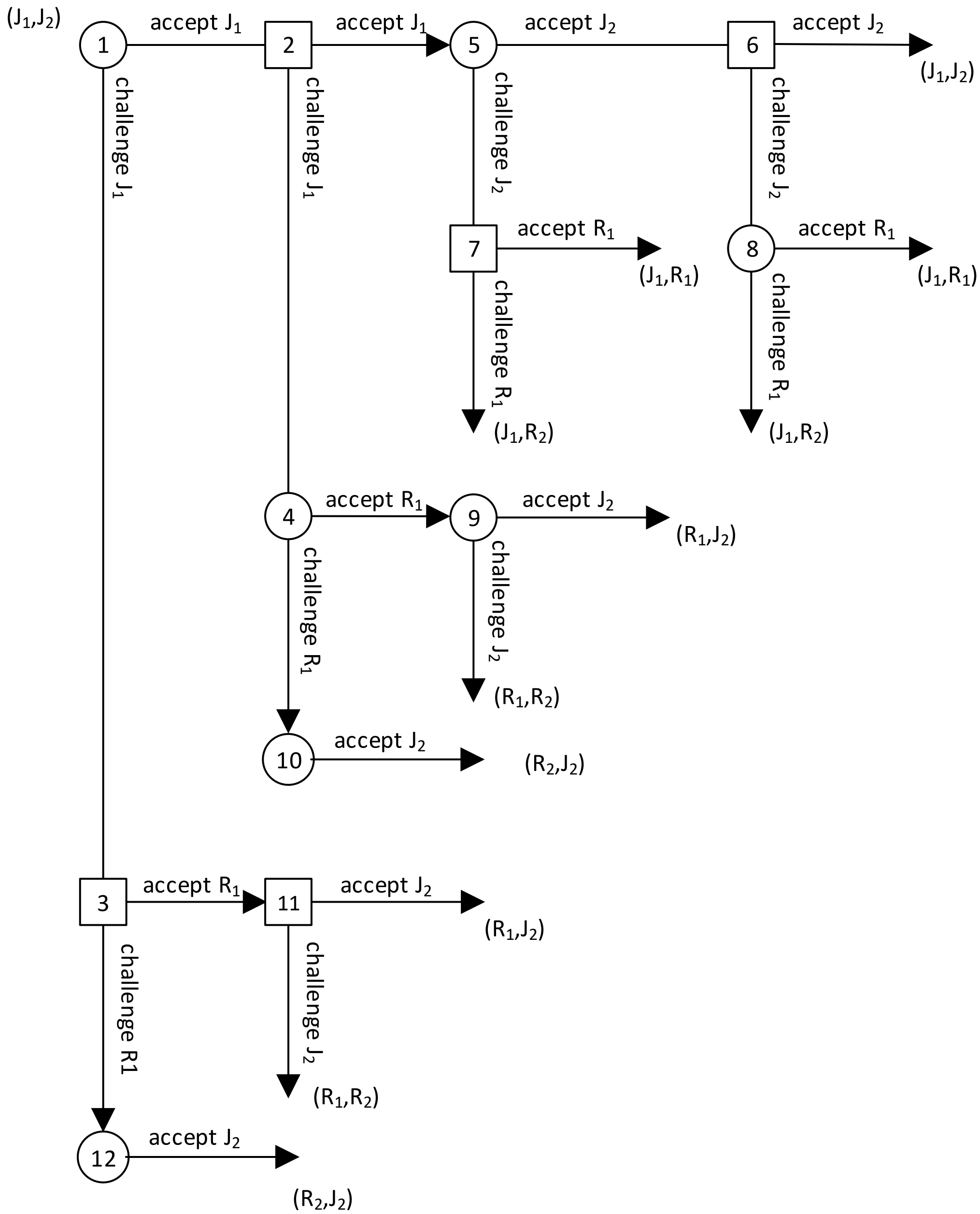

Figure 3. The complete game tree for the selection process described in Example 2. Arrows represent possible outcomes for each game.

We now determine under what conditions the prosecution should challenge the first juror, J_1. Using the results of Example 1, we find that the expected value of node 12 is $V_{12} = 0.5 \times 0.5 = 0.25$. The value for node 11 is $V_{11} = 0.375$. The defense challenge threshold for node 3 is therefore given by equation (8) as $r^* = 0.25/0.375 = 0.667$. The expected value of node 3 is

$$V_3 = 0.375 \times \int_0^{0.667} r\,dr + 0.25 \times \int_{0.667}^{1} dr = 0.167 \qquad (10)$$

Applying a similar procedure to the branch below node 2, we find the value V_4= 0.363. Again, applying the results of Example 1, we find the value of node 5 $V_5 = 0.5$.The game tree is thereby reduced to the tree shown in Figure 4.

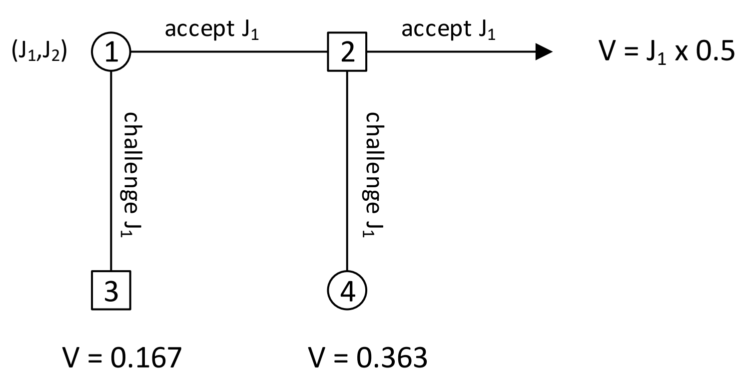

Figure 4. The game tree of Figure 3 reduced by recursive application of Equation (9)

Applying equation (8), we find the threshold values $r_* = 0.167/0.5 = 0.333$, and $r^* = 0.363/0.5 = 0.725$. The expected value of the jury prior to examination of any jurors is given by:

$$V_1 = 0.167 \times \int_0^{0.333} dr + 0.5 \times \int_{0.333}^{0.725} r\, dr + 0.363 \times \int_{0.725}^{1} dr = 0.259$$

10. Numerical Calculation of Challenge Thresholds

The number of decision points present in real-world jury selection scenarios preclude manual calculations such as those given above. Computer algorithms have been developed by the author to evaluate such scenarios. This section presents the results of computer-based numerical studies representing scenarios including up to 12 seats and each party having up to 6 peremptory challenges. The results presented assume a Strike and Replace selection system, separable utility functions obeying (2), and uniform jury pool distributions over a rating scale of 0 – 10. Care should be exercised when attempting to extrapolate the results given here to scenarios using different selection systems, different jury utility functions, and/or different jury pool rating distributions.

Results are reported as variation of challenge threshold values, r_* and r^*, as a function of state parameters. Figure 5 shows the variation of challenge thresholds with the number of peremptory challenges available to the prosecution and to the defense, keeping the jury size fixed at a single seat. As expected, the greater the number of challenges available to the prosecution, the larger the value of the challenge threshold, indicating that the prosecution is willing to accept greater risk in challenging a juror when possessing more challenges. Likewise, when the defense has a greater number of challenges (lower curves in Figure 5), the prosecution must be more conservative in the exercise of each challenge, thereby lowering the prosecution challenge threshold. We note that the challenge threshold varies over a significant portion of the rating range, in this case from approximately 2 to 8, for the parameter space sampled (1 to 6 challenges for the prosecution and 0 to 6 challenges for the defense). This indicates that game theory solutions to jury selection processes may differ significantly from intuitive or 'rule of thumb' approaches such as using a fixed challenge threshold set at the average value of the jury pool rating distribution.

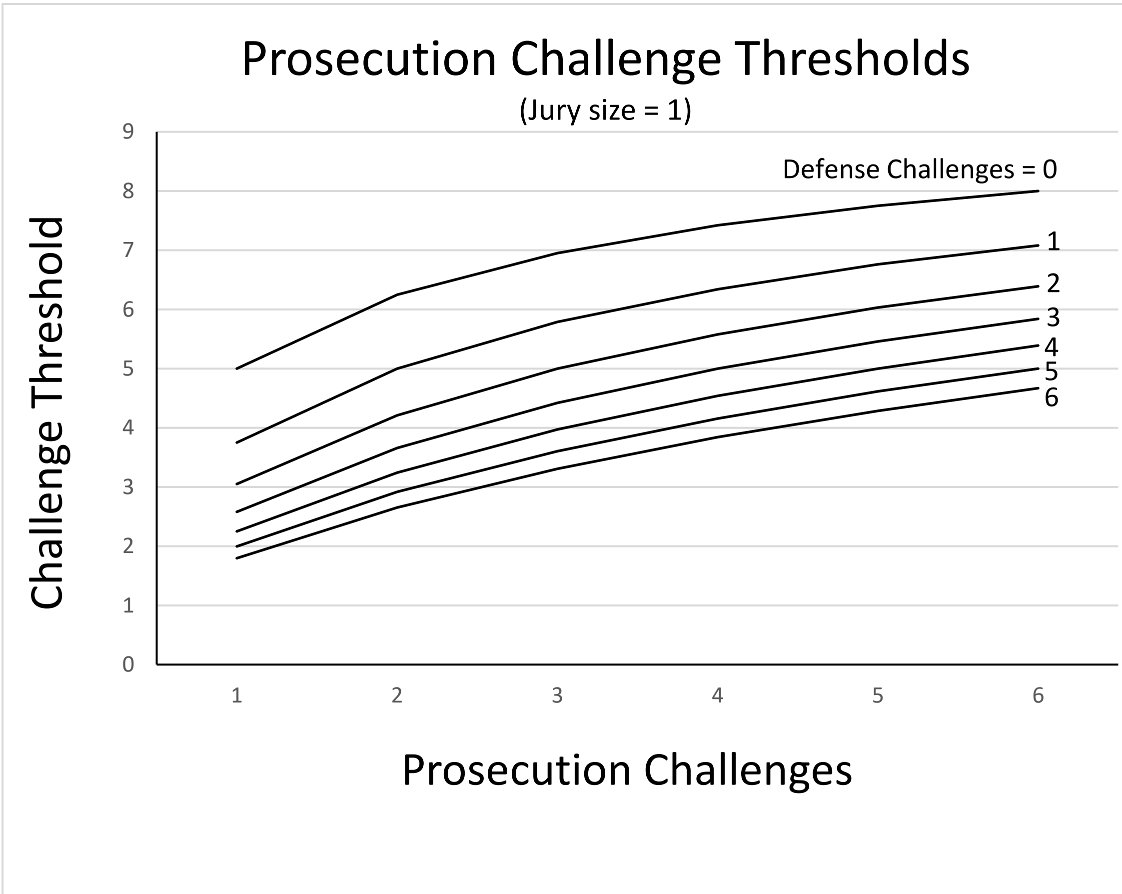

Figure 5. Numerically calculated prosecution challenge threshold values for a single seat jury. Different curves represent different numbers of peremptory challenges available to the defense.

11. Varying the Jury Size

Parties participating in a Strike and Replace or similar selection system would be expected to be more conservative in the exercise of peremptory challenges as the number of jury seats remaining to fill increases. All else being equal, a litigant with a single seat to fill would be expected to set a challenge threshold closer to pool average than the same litigant working to fill 2 jury seats. This is simply the result of spreading a fixed number of challenges over a greater number of jurors while accounting for the possibility of the appearance of an unfavorable juror after the exhaustion of available challenges. While the tendency to conserve challenges may be qualitatively intuitive, the degree to which optimal challenge thresholds are sensitive to varying jury seat numbers are only calculable within a mathematical framework such as the game theory models described here.

The game theory model for a Strike and Replace selection system with a uniform juror rating distribution on a scale of 0 to 10 and with both parties having a single available peremptory challenge are used to calculate the challenge thresholds for prosecution and defense as a function of the number of jury seats remaining to fill. These challenge thresholds are shown in Figure 6. Challenge thresholds are calculated assuming that the jury seats are considered in order and the party in question is the first to optionally exercise a challenge for each jury seat (i.e., prosecution thresholds are calculated assuming that the prosecution has the first option to challenge when each new jury seat is considered, and vice versa.).

Figure 6 is divided into three regions by the threshold curves: 1) the juror will be challenged by the defense, 2) both parties will accept the juror, and 3) the juror will be challenged by the prosecution. The region of acceptance becomes larger as the jury size increases, reflecting the risk aversion of the parties when required to exercise a fixed number of challenges over a larger number of jury seats.

We note the increasing departure of the game theory-based challenge thresholds from the pool average value as the jury size increases. For a jury of twelve seats, the defense challenge threshold is over 4 points higher than the pool average of 5.

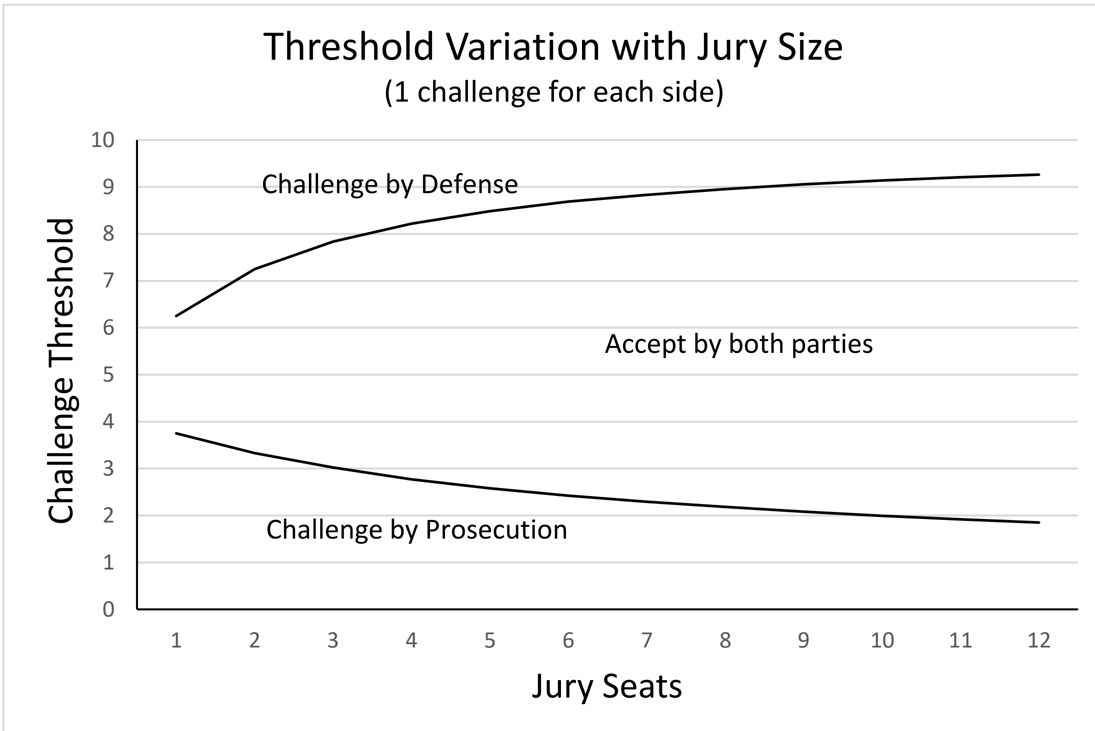

Figure 6. Challenge thresholds for multiple seat juries with each side having a single peremptory challenge.

12. Challenge Scenarios and Strategies

The relative performance of the game theory-based solutions presented here are determined by 'playing' such solutions against other possible strategies in computerized simulation studies. Due to the probabilistic nature of jury selection, no single instance of a selection game can determine the relative performance of a set of strategies. However, over large numbers of 'plays', meaningful results can be obtained. We define the following strategies for comparison studies:

Game Theory:	The strategy presented in this work
Pool Average:	Challenge above (below) the pool average for prosecution (defense)
Coin Toss:	Challenge at random, independent of juror rating.
Always accept:	Never exercise a challenge

Computer simulations of Strike and Replace scenarios have been implemented for various jury sizes and various numbers of peremptory challenges. Results for up to 12 jury seats with each side having 4 available peremptory challenges are shown in Figure 7. Here, the defense has been programmed to use Game Theory strategy whereas the prosecution variously uses Game Theory, Pool Average, Coin Toss, or Always Accept. The resulting jury values are the product of the individual juror ratings, normalized to a scale of 0 – 10. On this scale, prosecution and defense performing equally well in their selection strategies would result in a jury rating of 5. Each data point represents 1000 computer-simulated plays.

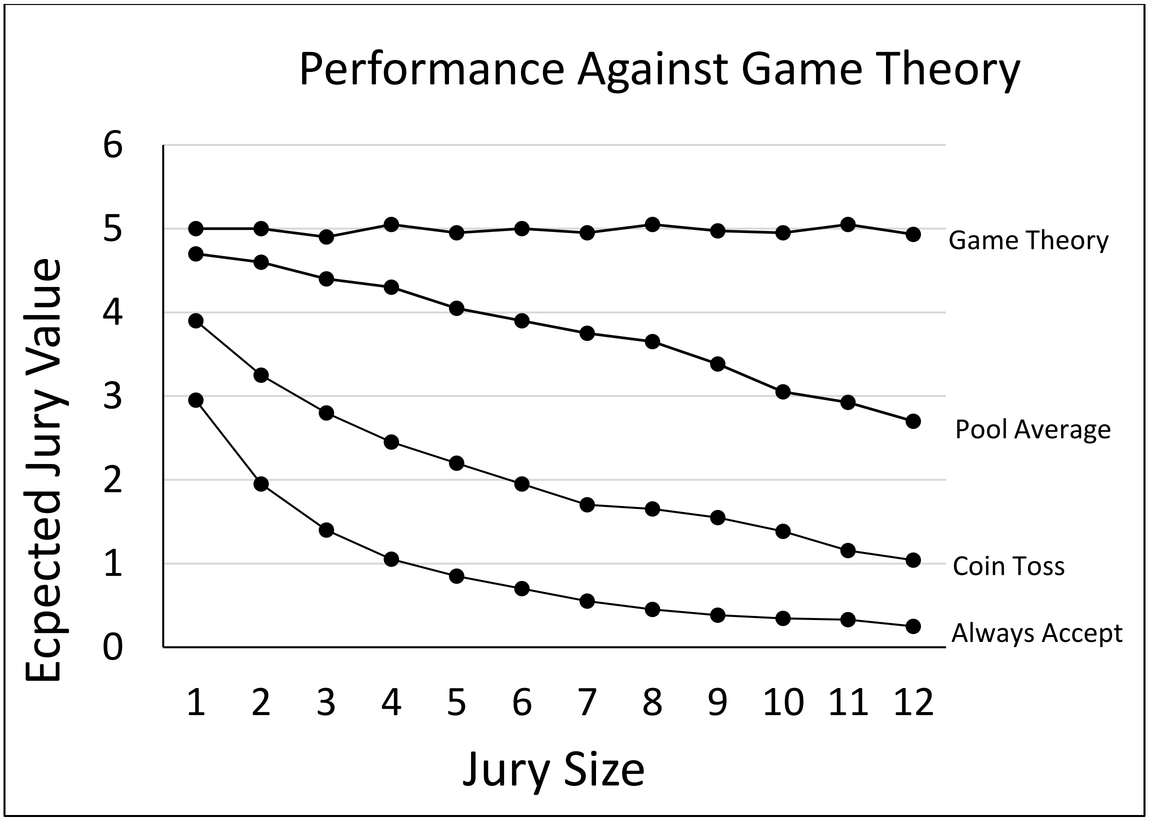

Figure 7. Results of simulation studies of Game Theory played against various strategies for various jury sizes. Each side has 4 available peremptory challenges.

As expected, the difference in performance between the various strategies becomes larger as the jury size increases, reflecting the greater uncertainties associated with larger juries, with a large effect for real-world jury sizes of 12 seats. Coin Toss and Always Accept, both of which make no use of juror rating information, are tantamount to obtaining low quality juror ratings, or simply not rating jurors at all. As expected, Coin Toss and Always Accept perform poorly against Game Theory, which can eliminate undesirable jurors. Pool Average which can eliminate the most undesirable jurors, performs considerably better than Coin Toss and Always Accept. Game Theory, however, takes account of potential replacement juror ratings as well as likely actions by the opposing party, and making use of this information performs better than Pool Average for realistic jury sizes.

13. Conclusions

Game theory based jury selection strategies are designed to maximize the probability of obtaining a favorable jury through the strategic use of peremptory challenges. We have shown that given a set of selection rules, a set juror ratings, and a set of jury pool rating distributions, the optimal strategic use of peremptory challenges is mathematically calculable. Two party zero-sum selection games of arbitrary jury size have been analyzed in the case of multiplicably-separable utility functions, with emphasis placed on actionable challenge threshold values.

We have presented numerical simulations which compare various strategies for the exercise of peremptory challenges, including Always Accept, Coin Toss, Pool Average and Game Theory. Always Accept and Coin Toss make no use of juror rating information since challenges are simply exercised at random or are not exercised at all. Pool Average makes use of juror rating information, and results in expected jury values higher than Coin Toss and Always Accept, as shown by the associated curves in Figure 7. Pool Average, however, it is not able to account for the likely actions of opposing parties and, as a result, is prone to seating jurors who may be less favorable than the ones who were challenged. The strategic exercise of peremptory challenges using Game Theory accounts for opposing party actions, and provides an additional advantage, as seen by the difference in Figure 7 between the Game Theory and Pool Average strategies. Our numerical studies show that for real-world jury selection scenarios, the advantage afforded by Game Theory is of the same order as the advantage afforded by rating jurors at all. This suggests that parties making investments in juror ratings through mock trials, demographic surveys, etc. would do well to make similar investments in the strategic exercise of peremptory challenges using Game Theory.

Experienced litigators and trial consultants may knowingly or intuitively apply game theoretic principles to the exercise of peremptory challenges. However, it would be impractical for them to determine precise challenge thresholds in the absence of mathematical calculations described here. Pool Average and other such 'rule of thumb' strategies commonly employed by attorneys and their consultants, may have the advantage of being simple to execute in a courtroom setting, however, such strategies are not expected to perform as well as game theory strategies on average. Computing power sufficient to perform game theoretic calculations in real-world jury selection scenarios is currently available in portable laptop computers and will likely soon be available in computer tablets, phablets and smart phones. We expect that computer-aided game theory-based jury selection will become commonplace in courtrooms in the near future.

Finally, the simulations presented here, based on the simplification of numerical juror ratings, do not consider the effects of juror-juror interactions. Such interactions may be common during juror deliberations. Future work incorporating such effects in a game theoretic analysis would be welcome.

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